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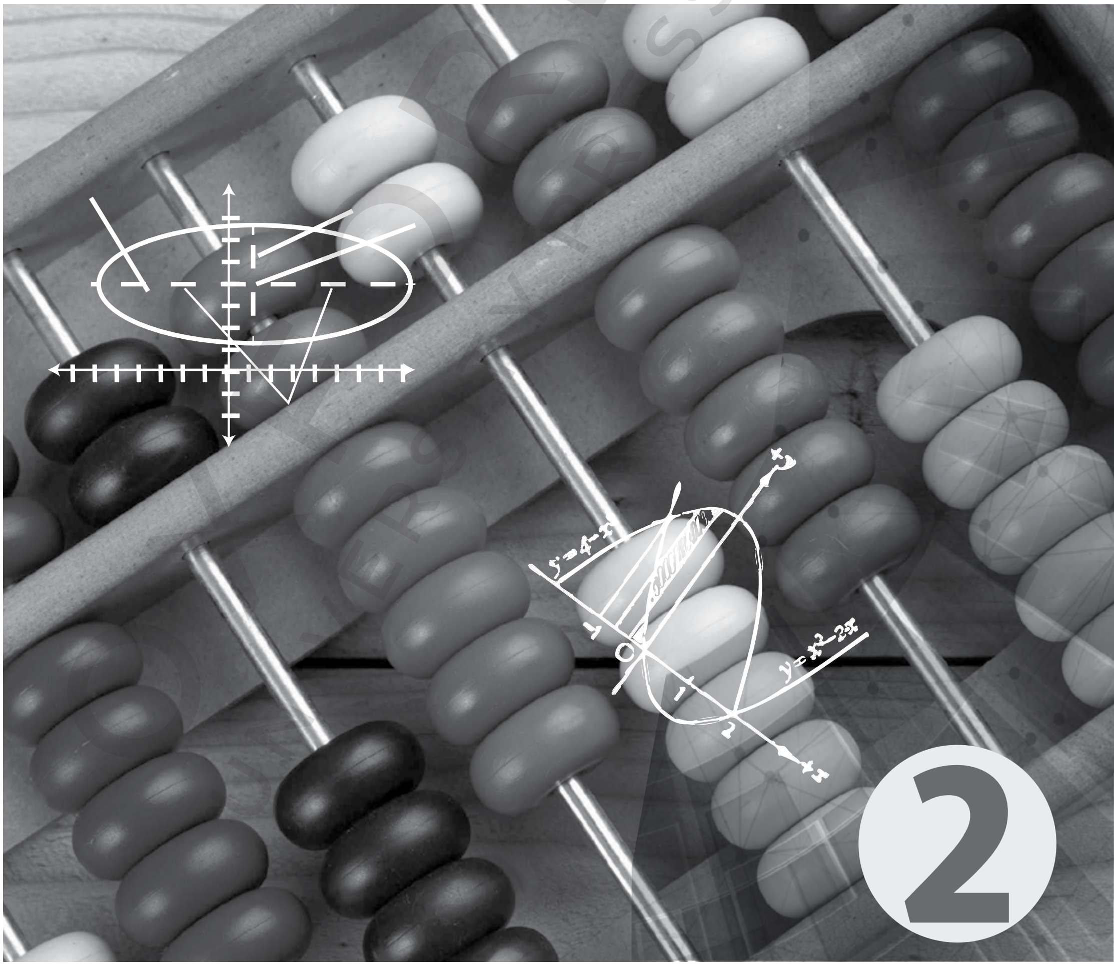
UPDATED
7th EDITION

NEW SYLLABUS MATHEMATICS WORKBOOK FULL SOLUTIONS

A Comprehensive Mathematics Programme for Grade 7



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OXFORD
UNIVERSITY PRESS

ANSWERS

Chapter 1 Real Numbers and Approximation

Basic

1. (a)

$$\begin{array}{r} 144.735 \\ \times \quad 0.15 \\ \hline 723675 \\ +144735 \\ \hline 21.71025 \\ \hline \end{array}$$

$\therefore$ 144.735 × 0.15 = 21.710 25

(b)

$$\begin{array}{r} 0.35 \\ \times \quad 0.096 \\ \hline 210 \\ +\ 315 \\ \hline 0.03360 \\ \hline \end{array}$$

$\therefore$ 0.35 × 0.096 = 0.0336

(c)

$$\begin{array}{r} 1.84 \\ \times \quad 0.092 \\ \hline 368 \\ +1656 \\ \hline 0.16928 \\ \hline \end{array}$$

$\therefore$ 1.84 × 0.092 = 0.169 28

(d) 4.86 ÷ 1.20

$= \dfrac{4.86}{1.20}$

$= \dfrac{486}{120}$

$$\begin{array}{r} 4.05 \\ 120\overline{)\,486} \\ -480 \\ \hline 60 \\ -\quad 0 \\ \hline 600 \\ -600 \\ \hline 0 \\ \hline \end{array}$$

$\therefore$ 486 ÷ 120 = 4.05

(e) 1.921 68 ÷ 62.8

$= \dfrac{1.92168}{62.8}$

$= \dfrac{19.2168}{628}$

$$\begin{array}{r} 0.0306 \\ 628\overline{)\,19.2168} \\ -0 \\ \hline 192 \\ -\quad 0 \\ \hline 1921 \\ -1884 \\ \hline 376 \\ -\quad 0 \\ \hline 3768 \\ -3768 \\ \hline 0 \\ \hline \end{array}$$

$\therefore$ 1.92168 ÷ 62.8 = 0.0306

(f) 0.003 48 ÷ 0.048

$= \dfrac{0.00348}{0.048}$

$= \dfrac{3.48}{48}$

$$\begin{array}{r} 0.0725 \\ 48\overline{)\,3.48} \\ -0 \\ \hline 34 \\ -\quad 0 \\ \hline 348 \\ -336 \\ \hline 120 \\ -\quad 96 \\ \hline 240 \\ -240 \\ \hline 0 \\ \hline \end{array}$$

$\therefore$ 0.003 48 ÷ 0.048 = 0.0725

2. (a) 5.3 – (–4.9)
= 5.3 + 4.9
= 10.2

(b) 3.3 + (–2.7)
= 3.3 – 2.7
= 0.6

(c) –15.4 + 8.9
= –(15.4 – 8.9)
= –6.5

(d) $-17.3 - 6.25$
$= -(17.3 + 6.25)$
$= -23.55$

3. (a) 789 500 (to the nearest 100)
(b) 790 000 (to the nearest 1000)
(c) 790 000 (to the nearest 10 000)

4. (a) 2.5 (to 1 d.p.)
(b) 18.5 (to 1 d.p.)
(c) 36.1 (to 1 d.p.)
(d) 138.1 (to 1 d.p.)

5. (a) 4.70 (to 2 d.p.)
(b) 14.94 (to 2 d.p.)
(c) 26.80 (to 2 d.p.)
(d) 0.05 (to 2 d.p.)

6. (a) 4.826 (to 3 d.p.)
(b) 6.828 (to 3 d.p.)
(c) 7.450 (to 3 d.p.)
(d) 8.445 (to 3 d.p.)
(e) 11.639 (to 3 d.p.)
(f) 13.451 (to 3 d.p.)
(g) 32.929 (to 3 d.p.)
(h) 0.038 (to 3 d.p.)

7. (a) 36.3 (to 1 d.p.)
(b) 36 (to the nearest whole number)
(c) 36.260 (to 3 d.p.)

8. (a) 984.61 (to 2 d.p.)
(b) 984.6 (to 4 s.f.)
(c) 984.608 (to 3 d.p.)
(d) 984.61 (to the nearest hundredth)

9. (a) 14.0 kg (to the nearest 0.1 kg)
(b) 57.5 kg (to the nearest 0.1 kg)
(c) 108.4 kg (to the nearest 0.1 kg)
(d) 763.2 kg (to the nearest 0.1 kg)

10. (a) 7.0 cm^2 (to the nearest $\frac{1}{10}$ cm^2)
(b) 40.1 cm^2 (to the nearest $\frac{1}{10}$ cm^2)
(c) 148.3 cm^2 (to the nearest $\frac{1}{10}$ cm^2)
(d) 168.4 cm^2 (to the nearest $\frac{1}{10}$ cm^2)

11. (a) 5620 km (to the nearest 10 km)
(b) 900 cm (to the nearest 100 cm)
(c) 2.45 g (to the nearest $\frac{1}{100}$ g)
(d) PKR 50 000 (to the nearest PKR 10 000)

12. The calculation is $297 \div 19.91$.
$297 \div 19.91$
$\approx 300 \div 20$
$= 15$ (to 2 s.f.)
15 litres of petrol is used to travel 1 km.

Intermediate

13. (a) $\frac{0.25}{0.05} \times \left(-\frac{0.18}{1.3}\right)$
$= \frac{25}{5} \times \left(\frac{-1.8}{13}\right)$
$= 5 \times \left(\frac{-1.8}{13}\right)$
$= -\frac{9}{13}$

(b) $\frac{0.0064}{0.04} \times \left(-\frac{1.8}{0.16}\right)$
$= \frac{0.64}{4} \times \left(-\frac{180}{16}\right)$
$= 0.16 \times \left(-\frac{45}{4}\right)$
$= -1.8$

(c) $(-0.3)^2 \times \left(\frac{-1.4}{0.07}\right) - 0.78$
$= \left(-\frac{3}{10}\right)^2 \times \left(\frac{-140}{7}\right) - 0.78$
$= \left(\frac{9}{100}\right) \times (-20) - 0.78$
$= -1.8 - 0.78$
$= -2.58$

(d) $(-0.4)^3 \times \left(\frac{-3.3}{0.11}\right) + 0.123$
$= \left(-\frac{4}{10}\right)^3 \times \left(\frac{-3.3}{0.11}\right) + 0.123$
$= \left(-\frac{64}{1000}\right) \times \left(\frac{-330}{11}\right) + 0.123$
$= \left(-\frac{64}{1000}\right) \times (-30) + 0.123$
$= 1.92 + 0.123$
$= 2.043$

14.

Reservoir	A	B	C	D
Water level	$-2 + 6 + 8$ $= 12$	$+1 + 3 - 7$ $= -3$	$-3 - 1 - 2$ $= -6$	$-5 + 9 - 1$ $= 3$

(a) Reservoir **A** caught the most rain.
(b) Reservoir **C** caught the least rain.
(c) Reservoir **D** because $3 > -3$.

15. Let the length of shorter piece of rope be x m.

Therefore, length of the longer piece of rope = $\frac{5}{4}x$ m.

$$x + \frac{5}{4}x = 6.3$$
$$\frac{4}{4}x + \frac{5}{4}x = 6.3$$
$$\frac{9}{4}x = 6.3$$
$$x = 6.3 \times \frac{4}{9}$$
$$x = 2.8$$

$\therefore$ Length of the shorter piece of rope is 2.8 m

16. Number of cups of flour Sarah used

$= \left(2\frac{1}{2} \times 9\right) + \left(2\frac{3}{4} \times 3\right)$

$= \left(\frac{5}{2} \times 9\right) + \left(\frac{11}{4} \times 3\right)$

$= \left(\frac{45}{2}\right) + \left(\frac{33}{4}\right)$

$= \frac{90}{4} + \frac{33}{4}$

$= \frac{123}{4}$

$= 30\frac{3}{4}$

17. Number of students in class A

$= \frac{4}{19} \times 247$

$= 52$

Number of students in class that travel to school by bus

$= \left(\frac{8}{13} \times 52\right) + 7$

$= 32 + 7$

$= 39$

Therefore, number of students in class A who do not travel by bus

$= 52 - 39$

$= 13$

18. (i) Fraction of cost price of refrigerator that Anusha pays

$= 1 - \frac{3}{10} - \frac{9}{20}$

$= \frac{20}{20} - \frac{6}{20} - \frac{9}{20}$

$= \frac{5}{20}$

$= \frac{1}{4}$

(ii) Cost of refrigerator

$= \text{PKR } 525 \div \frac{1}{4}$

$= \text{PKR } 525 \times 4$

$= \text{PKR } 2100$

Advanced

19. $$\left\{\left(5\frac{3}{4} - 2\frac{4}{7} + \left(-\frac{33}{8}\right) - \left(-\frac{1}{56}\right)\right)^2 - \sqrt[3]{\frac{8}{216}} \times \left[-\sqrt{\frac{3}{7}} - (-0.875)^3\right]\right\}$$
$$\div \left(\frac{\sqrt{35} - \sqrt[3]{29}}{-\tilde{\ } + \sqrt{-1\frac{1}{2} \times \left(-\frac{5}{8}\right)}}\right) = 0.866 \text{ (to 3 d.p.)}$$

Chapter 2 Direct and Inverse Proportions

Basic

1. Cost of 15 l of petrol $= \dfrac{\text{PKR } 14.70}{7} \times 15$

$= \text{PKR } 31.50$

2. Time taken for 1 tap to fill the bath tub $= 15 \times 2$

$= 30$ minutes

Time taken for 3 taps to fill the bath tub $= \dfrac{30}{3}$

$= 10$ minutes

Intermediate

3. **(i)** $a = kb$

When $b = 15$, $a = 75$,

$75 = k(15)$

$k = \dfrac{75}{15}$

$= 5$

$\therefore a = 5b$

When $b = 37.5$,

$a = 5(37.5)$

$= 187.5$

(ii) When $a = 195$,

$195 = 5b$

$b = \dfrac{195}{5}$

$= 39$

4. $w = kt$

When $t = 0.3$, $w = 1.8$,

$1.8 = k(0.3)$

$k = \dfrac{1.8}{0.3}$

$= 6$

$\therefore w = 6t$

When $t = 2.5$,

$w = 6(2.5)$

$= 15$

$\therefore$ 15 g of silver will be deposited.

5. $H = kd^3$

When $d = 6$, $H = 120$,

$120 = k(6)^3$

$= 216k$

$k = \dfrac{120}{216}$

$= \dfrac{5}{9}$

$\therefore H = \dfrac{5}{9} d^3$

When $d = 9$,

$H = \dfrac{5}{9}(9)^3$

$= 405$

$\therefore$ The shaft can transmit 405 horsepower.

6. Number of workers to complete in 1 day $= 6 \times 8$

$= 48$

Number of workers to complete in 12 days $= \dfrac{48}{12}$

$= 4$

Chapter 3 Application of Mathematics in Practical Situations

Basic

1. **(i)** $\frac{\text{Profit}}{\text{Original price}} \times 100\% = 15\%$

$\frac{150}{\text{Original price}} \times 100\% = 15\%$

Original price $= \frac{150 \times 100}{15}$ = PKR 1000

(ii) Profit = selling price – cost price

150 = selling price – 1000

Selling price of watch = 150 + 1000 = PKR 1150

2. **(i)** $\frac{\text{Discount}}{\text{Marked price}} \times 100\% = 8\%$

$\frac{112}{\text{Marked price}} \times 100\% = 8\%$

Marked price $= \frac{112 \times 100}{8}$ = PKR 1400

(ii) Discount = marked price – sale price

PKR 112 = PKR 1400 – sale price

Sale price = PKR 1400 – PKR 112 = PKR 1288

3. Selling price of the iPads = 900 × 15

= PKR 13 500

Increase = PKR 13 500 – PKR 10 000

= PKR 3500

(i) Percentage increase $= \frac{\text{Increase}}{\text{Cost price}} \times 100\%$

$= \frac{3500}{10\,000} \times 100\%$

= 35%

(ii) Percentage increase $= \frac{\text{Increase}}{\text{Selling price}} \times 100\%$

$= \frac{3500}{13\,500} \times 100\%$

= 25.9% (to 3 s.f.)

4. 107% of the marked price = PKR 27.20

$\frac{107}{100} \times$ marked price = 27.20

Marked price $= 27.20 \div \frac{107}{100}$

$= 27.20 \times \frac{100}{107}$

= PKR 25.42

5. **(a)** Amount of commission he receives

= 15% of PKR 50 000

$= \frac{15}{100} \times 50\,000$

= PKR 7500

(b) Let PKR x be the signing bonus.

15% of PKR x = PKR 4800

$\frac{15}{100} \times x = 4800$

$x = 4800 \div \frac{15}{100}$

$= 4800 \times \frac{100}{15}$

= 32 000

The amount of signing bonus is PKR 32 000.

Intermediate

6. **(i)** Percentage point is the difference between two percentages.

Percentage point of candidate C and A = 42 – 7

= 35%

Percentage point of candidate B and A = 39 – 7

= 32%

(ii) *To find the total number of voters,*

Method 1

Number of people who did not vote

= 20% of 15 000

$= \frac{20}{100} \times 15\,000$

= 3000

Number of people who voted

= 15 000 – 3000

= 12 000

Method 2

Percentage of people who voted

= 100% – 20% = 80%

Number of people who voted

= 80% of 15 000

$= \frac{80}{100} \times 15\,000$

= 12 000

To find the number of votes for each candidate

Number of people who voted for candidate A

= 7% of 12 000

$= \frac{7}{100} \times 12\,000$

= 840

Number of people who voted for candidate B
$= 39\%$ of 12 000
$= \frac{39}{100} \times 12\,000$
$= 4680$
Number of people who voted for candidate C
$= 42\%$ of 12 000
$= \frac{42}{100} \times 12\,000$
$= 5040$

7. Subscription + service charge
$= 110\%$ of PKR 59.90
$= \frac{110}{100} \times 59.90$
= PKR 65.89
Amount payable before GST
$= 113\%$ of PKR 65.89
= PKR 74.4557
Total cost of the bill
$= 107\%$ of PKR 74.4557
$= \frac{107}{100} \times$ PKR 74.4557
= PKR 79.67 (to the nearest paisa)

8. Original price of the coffee $= 3 \times 9 + 1 \times 13$
= PKR 40
Selling price of the mixture of coffee $= \frac{1.25}{0.1} \times 4$
= PKR 50
Profit $= 50 - 40 =$ PKR 10
Percentage profit $= \frac{10}{50} \times 100\%$
$= 20\%$

9. Original cost of tea $= 30 \times 32 + 20 \times 35$
= PKR 1660
Selling price of tea $= 40 \times (30 + 20)$
= PKR 2000
Profit $= 2000 - 1660 =$ PKR 340
Percentage profit $= \frac{340}{1660} \times 100\%$
$= 20.5\%$ (to 3 s.f.)

10. Price of an item after 8% discount
$= 92\%$ of PKR 45
$= \frac{92}{100} \times 45$
= PKR 41.40
Price of an item after a further discount of 9%
$= 91\%$ of PKR 41.40
$= \frac{91}{100} \times 41.40$
= PKR 37.67
She paid PKR 37.67 for the item.

11. 90% of the price which Junaid paid for the camera
= PKR 414
Price Junaid paid for the camera $= 414 \div \frac{90}{100}$
$= 414 \times \frac{100}{90}$
= PKR 460
115% of the original price of the camera = PKR 460
Original price of the camera $= 460 \div \frac{115}{100}$
$= 460 \times \frac{100}{115}$
= PKR 400
The original price of the camera is PKR 400.

12. Let the number of peaches be x.
Cost price of 1 peach = PKR $\frac{294}{x}$
Selling price of 1 peach
$= 140\%$ of PKR $\frac{294}{x}$
$= \frac{140}{100} \times \frac{294}{x}$
= PKR $\frac{411.6}{x}$
Amount collected from selling all the good peaches
$= 294 + 84 =$ PKR 378
$(x - 16) \times \frac{411.6}{x} = 378$
$411.6(x - 16) = 378x$
$411.6x - 378x = 6585.6$
$33.6x = 6585.6$
$x = 196$
$\therefore$ Majeed bought 196 peaches.

13. **(a)** $116\frac{2}{3}\%$ of the marked price = PKR 420
1% of the marked price $= \frac{420}{116\frac{2}{3}}$
100% of the marked price $= \frac{420}{116\frac{2}{3}} \times 100$
= PKR 360
The price paid by Tanveer is PKR 360.

(b) (i) Selling price of the display set

$= \left(100 - 10\frac{1}{2}\right)\%$ of PKR 420

$= 89\frac{1}{2}\%$ of PKR 420

$= \frac{179}{2}\%$ of PKR 420

$= \left(\frac{179}{2} \div 100\right) \times 420$

$= \frac{179}{2} \times \frac{1}{100} \times 420$

= PKR 375.90

The selling price of the display set is PKR 375.90.

(ii) Percentage profit $= \frac{375.90 - 360}{360} \times 100\%$

= 4.42% (to 3 s.f.)

14. Amount of commission the salesman got

= 25% of PKR 5264

$= \frac{25}{100} \times 5264$

= PKR 1316

Total income = basic salary + commission

= 520 + 1316

= PKR 1836

15. Total cost of the materials for building the fence without discount and goods tax

$= 5 \times 25 + 6 \times 12 + 1 \times 10 + 12 \times \frac{15}{6} + 300 \times \frac{10}{1000}$

= 125 + 72 + 10 + 30 + 3

= PKR 240

Cost of the materials after discount = 90% of 240

$= \frac{90}{100} \times 240$

= PKR 216

Cost of the materials with goods tax

= 115% of PKR 216

$= \frac{115}{100} \times 216$

= PKR 248.40

The total amount that he has to pay, after discount and goods tax, is PKR 248.40.

16. (i) Total reliefs

= PKR 1000 + PKR 2000 + PKR 5000 + PKR 4500 + PKR 6000

= PKR 18 500

Taxable income = PKR 56 000 – PKR 18 500

= PKR 37 500

(ii)

Tax

PKR 37 500 → First PKR 30 000 : PKR 200

PKR 37 500 → Next PKR 7500 : 3.5% of PKR 7500 = PKR 262.50

∴ Income tax payable = PKR 200 + PKR 262.50

= PKR 462.50

(iii) Percentage of tax $= \frac{\text{PKR } 462.50}{\text{PKR } 37\,500} \times 100\%$

= 1.23% (to 3 s.f.)

17. (i) Commission earned for selling the HDB four-room flat

$= \left(\text{PKR } 15\,000 \times \frac{5}{100}\right) + \left(\text{PKR } 45\,000 \times \frac{3}{100}\right) + \left(\text{PKR } 40\,000 \times \frac{2.5}{100}\right) + (\text{PKR } 58\,500$

$- \text{PKR } 15\,000$

$- \text{PKR } 45\,000 - \text{PKR } 40\,000) \times \frac{2}{100}$

= PKR 12 800

Commission earned for selling a private house

$= \left(\text{PKR } 15\,000 \times \frac{5}{100}\right) + \left(\text{PKR } 45\,000 \times \frac{3}{100}\right) + \left(\text{PKR } 40\,000 \times \frac{2.5}{100}\right) + (\text{PKR } 1\,085\,000 -$

$\text{PKR } 15\,000 - \text{PKR } 45\,000 - \text{PKR } 40\,000) \times \frac{2}{100}$

= PKR 22 800

(ii) Total commission received

= PKR 22 800 + PKR 12 800

= PKR 35 600

Advanced

18. (i) Selling price of the condominium

= 90% of PKR 950 000

$= \frac{90}{100} \times 950\,000$

= PKR 855 000

(ii) Amount Manal received after paying the agent

= 98% of PKR 855 000

$= \frac{98}{100} \times 855\,000$

= PKR 837 900

(iii) Amount agent received from seller

= PKR 855 000 – PKR 837 900

= PKR 17 100

Amount agent received from buyer

= 5% of PKR 855 000

$= \frac{5}{100} \times 855\ 000$

= PKR 42 750

Total amount received by the agent

= 42 750 + 17 100

= PKR 59 850

19. (i) Number of litres used $= \frac{\text{PKR } 3600}{\text{PKR } 2.00} = 1800$ litres

(ii) Total distance travelled = 1800 × 16

= 28 800 km

(iii) Total cost in 2011

= PKR 3600 + PKR 2000 + PKR 850 + PKR 880

= PKR 7330

(iv) Total cost in 2012

$= \text{PKR } 880 + \left(\text{PKR } 3600 \times \frac{100 + 5}{100}\right)$

$+ \left(\text{PKR } 850 \times \frac{100 + 15}{100}\right) + \left(\text{PKR } 2000 \times \frac{100 - 10}{100}\right)$

= PKR 880 + PKR 3780 + PKR 977.50 + PKR 1800

= PKR 7437.50

Increase = PKR 7437.50 – PKR 7330 = PKR 107.50

Percentage increase $= \frac{\text{PKR } 107.50}{\text{PKR } 7330} \times 100\%$

= 1.5% (to 2 s.f.)

New Trend

20. Let the original price of the toy be PKR x.

180% of $x = 900$

$\frac{180}{100} \times x = 900$

$x = 900 \div \frac{180}{100}$

$= 900 \times \frac{100}{180}$

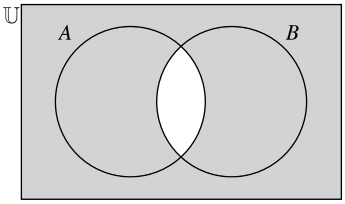
$= 500$

The original price of the toy is PKR 500.

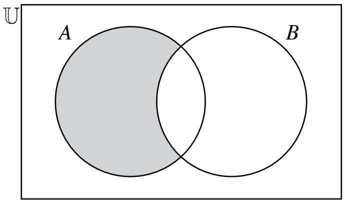
Chapter 4 Sets

Basic

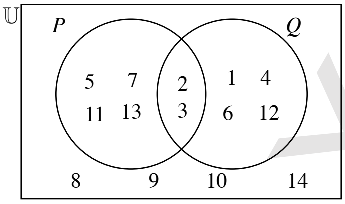
1. **(a)** $A \cup B' = \{a, b, c, x, y, m, n\}$
   (b) $A' \cap B' = \{m, n\}$
   (c) $A \cap B' = \{a, b, c\}$
2. **(a)** $A \cup B' = \{1, 2, 3, 4, 5, 7, 8\}$
   (b) $A' \cap B' = \{4, 8\}$
   (c) $A \cap B' = \{1, 2, 7\}$
3. **(a)** $A' \cup B'$



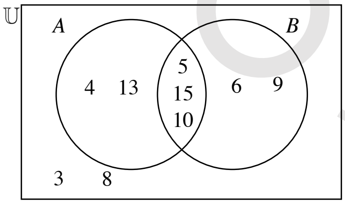
   (b) $A \cap B'$



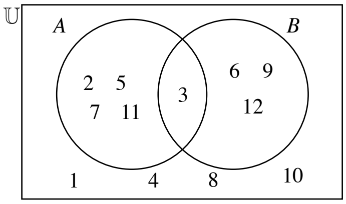
4. $\mathbb{U} = \{x : x \text{ is an integer}, 1 \leqslant x \leqslant 14\} = \{1, 2, 3, \ldots, 13, 14\}$
   $P = \{x : x \text{ is a prime number}\} = \{2, 3, 5, 7, 11, 13\}$
   $Q = \{x : x \text{ is a factor of } 12\} = \{1, 2, 3, 4, 6, 12\}$
   (a)



   (b) **(i)** $P \cup Q' = \{2, 3, 5, 7, 8, 9, 10, 11, 13, 14\}$
   (ii) $P' \cap Q' = \{8, 9, 10, 14\}$
5. **(a)**



   (b) **(i)** $(A \cap B)' = \{3, 4, 6, 8, 9, 13\}$
   (ii) $A' \cap B = \{6, 9\}$
6. $\mathbb{U} = \{x : x \text{ is an integer}, 1 \leqslant x \leqslant 12\} = \{1, 2, 3, \ldots, 11, 12\}$
   $A = \{x : x \text{ is a prime number}\} = \{2, 3, 5, 7, 11\}$
   $B = \{x : x \text{ is a multiple of } 3\} = \{3, 6, 9, 12\}$
   (a)



   (b) $A \cap B' = \{2, 5, 7, 11\}$
7. **(a)** $B \cap A'$
   (b) B'

Intermediate

8. **(a)** $A \cap B = \{e, x\}$
   (b) $A \cup C' = \{a, b, e, d, x, h, m, y, z\}$
   (c) $B \cup A' = \{e, x, h, m, n, k, y, z\}$
   (d) $B' \cap C' = \{a, b, y, z\}$
   (e) $A \cap B \cup C = \{e, x, d, k, n\}$
9. $\mathbb{U} = \{\text{polygons}\}$
   $A = \{\text{quadrilaterals}\}$
   $B = \{\text{regular polygons}\}$
   (a) square or rhombus
   (b) rectangle or parallelogram
10. $\mathbb{U} = \{x : x \text{ is an integer}, 12 \leqslant x \leqslant 39\}$
    $= \{12, 13, 14, \ldots, 37, 38, 39\}$
    $A = \{x : x \text{ is a multiple of } 5\} = \{15, 20, 25, 30, 35\}$
    $B = \{x : x \text{ is a perfect square}\} = \{16, 25, 36\}$
    $C = \{x : x \text{ is odd}\} = \{13, 15, 17, \ldots, 35, 37, 39\}$
    (a) $A \cap B = \{25\}$
    (b) $A \cap C = \{15, 25, 35\}$
    (c) $B \cup C = \{13, 15, 16, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 36, 37, 39\}$
11. $\mathbb{U} = \{x : x \text{ is an integer}\}$
    $A = \{x : x > 4\}$
    $B = \{x : -1 < x \leqslant 10\}$
    $C = \{x : x < 8\}$
    (a) $A \cap B = \{x : 4 < x \leqslant 10\}$
    (b) $B \cap C = \{x : -1 < x < 8\}$
    (c) $A' \cap B = \{x : -1 < x \leqslant 4\}$
    (d) $A' \cap C = \{x : x \leqslant 4\}$

12. c

13. $\mathbb{U} = \{x : x$ is whole number and $x \leqslant 20\}$

$A = \{2, 4, 6, 8, 10, 12\}$

$B = \{1, 4, 9, 16\}$

(a) $A \cap B' = \{2, 6, 8, 10, 12\}$

(b) $A' \cap B = \{1, 9, 16\}$

(c) $A' \cap B' = \{3, 5, 7, 11, 13, 14, 15, 17, 18, 19, 20\}$

(d) $A' \cup B' = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

14. $\mathbb{U} = \{(x, y) : x$ and y are integers$\}$

$P = \{(x, y) : 0 < x \leqslant 3$ and $0 \leqslant y < 6\}$

$Q = \{(x, y) : 2 \leqslant x < 8$ and $5 \leqslant y \leqslant 9\}$

$P \cap Q = \{(x, y) : 2 \leqslant x \leqslant 3$ and $5 \leqslant y < 6\}$

$x = 2, 3$ and $y = 5$

$\therefore P \cap Q = \{(2, 5), (3, 5)\}$

15. $\mathbb{U} = \{$a, b, c, d, e, f, g$\}$

$A = \{$a, c, f, g$\}$

$B = \{$a, c, g$\}$

$C = \{$b, c, e, f$\}$

(i) $(A \cap B)' = \{$b, d, e, f$\}$

(ii) $A \cup C' = \{$a, c, d, f, g$\}$

16. $\mathbb{U} = \{$all triangles$\}$

$A = \{$isosceles triangles$\}$

$B = \{$equilateral triangles$\}$

$C = \{$right-angled triangles$\}$

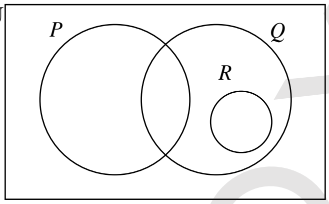
(a) $A \cup B = A$

(b) $B \cap C = \varnothing$

(c) $A \cap B = B$

17. **(a)** $A = B$

(b) $\mathbb{U}$



18. $\mathbb{U} = \{x : x$ is an integer$\}$

$A = \{x : 20 < x \leqslant 32\}$

$B = \{x : 24 \leqslant x \leqslant 37\}$

(a) $A \cap B = \{x : 24 \leqslant x \leqslant 32\}$
$= \{24, 25, 26, 27, 28, 29, 30, 31, 32\}$

(b) $A \cup B = \{x : 20 < x \leqslant 37\}$
$= \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37\}$

19. $\mathbb{U} = \{x : x$ is an integer, $4 \leqslant x \leqslant 22\} = \{4, 5, 6, \ldots, 22\}$

$A = \{x : x$ is a multiple of $5\} = \{5, 10, 15, 20\}$

$B = \{x : x$ is a prime number$\} = \{5, 7, 11, 13, 17, 19\}$

$C = \{x : x$ is a factor of $30\} = \{5, 6, 10, 15\}$

(a) $A \cup C = \{5, 6, 10, 15, 20\}$

(b) $B \cap C = \{5\}$

20. $\mathbb{U} = \{6, 8, 10, 12, 13, 14, 15, 16, 18, 20, 21\}$

$A = \{x : x$ is a multiple of $3\} = \{6, 12, 15, 18, 21\}$

$B = \{x : 2x < 33\} = \{6, 8, 10, 12, 13, 14, 15, 16\}$

$A \cup B = \{6, 8, 10, 12, 13, 14, 15, 16, 18, 21\}$

21. $\mathbb{U} = \{x : x$ is a natural number, $2 \leqslant x \leqslant 15\}$
$= \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

$A = \{x : x$ is a multiple of $3\} = \{3, 6, 9, 12, 15\}$

$B = \{x : x$ is even$\} = \{2, 4, 6, 8, 10, 12, 14\}$

$A' \cap B = \{2, 4, 8, 10, 14\}$

22. $\mathbb{U} = \{x : x$ is a positive integer$\}$

$A = \{x : 7 < 3x < 28\} = \{3, 4, 5, 6, 7, 8, 9\}$

$B = \{x : 3 < 2x + 1 < 25\} = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$C = \{x : 1 < \frac{x}{2} \leqslant 9\}$
$= \{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$

23. **(a)** $A' \cap B = B$

(b) $A \cup B' = B'$

24. $\mathbb{U} = \{x : x$ is a positive integer and $20 \leqslant x \leqslant 90\}$

$A = \{x : x$ is a multiple of $3\}$
$= \{21, 24, 27, 30, 33, 36, ..., 90\}$

$B = \{x : x$ is a perfect square$\} = \{25, 36, 49, 64, 81\}$

$C = \{x :$ unit digit of x is $1\} = \{21, 31, 41, 51, 61, 71, 81\}$

(i) $A \cap B = \{36, 81\}$

(ii) $A \cap C = \{21, 51, 81\}$

25. $\mathbb{U} = \{x : x$ is a positive integer and $0 \leqslant x \leqslant 24\}$

$A = \{x : x$ is a prime number$\}$
$= \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$

$B = \{x : 12 < 3x < 37\} = \{5, 6, 7, 8, 9, 10, 11, 12\}$

$A \cap B = \{5, 7, 11\}$

26. **(a)** $P \cup Q = P$

(b) $Q \cap P' = \varnothing$

27. **(a)** $A \cap B = A$

(b) $A \cup B = B$

28. $\mathbb{U} = \{$integers$\}$

$A = \{$factors of $4\} = \{1, 2, 4\}$

$B = \{$factors of $6\} = \{1, 2, 3, 6\}$

$C = \{$factors of $12\} = \{1, 2, 3, 4, 6, 12\}$

$D = \{$factors of $9\} = \{1, 3, 9\}$

(a) $A \cup B = \{1, 2, 3, 4, 6\}$

(b) $B \cap C = \{1, 2, 3, 6\}$

(c) $C \cap D = \{1, 3\}$

Advanced

29. $\mathbb{U}$ = {polygons}

A = {polygons with all sides equal}

B = {polygons with all angles equal}

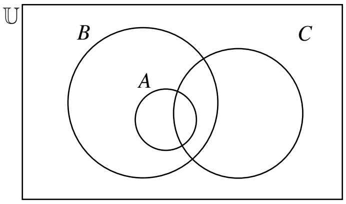
C = {triangles}

D = {quadrilaterals}

(a) $A \cap C$ = equilateral triangle

(b) $A \cap D$ = rhombus

(c) $B \cap D$ = square or rectangle

30. 

31. $\mathbb{U} = \{x : x$ is an integer less than 22$\}$

$A = \{x : x$ is a prime number less than 20$\}$

$= \{2, 3, 5, 7, 11, 13, 17, 19\}$

$B = \{x : a < x < b\}$

For $A \cap B = \varnothing$,

$8 < x < 10$ or $14 < x < 16$

$\therefore a = 8, b = 10$ or $a = 14, b = 16$.

32. $A = \{(x, y) : x + y = 4\}$

$B = \{(x, y) : x = 2\}$

$C = \{(x, y) : y = 2x\}$

(a) $A \cap B = \{(x, y) : x = 2, y = 2\} = \{(2, 2)\}$

(b) $B \cap C = \{(x, y) : x = 2, y = 4\} = \{(2, 4)\}$

(c) $A \cap C = \{(x, y) : x + y = 4, y = 2x\}$

$$= \left\{(x, y) : x = 1\frac{1}{3},\ y = 2\right\}$$

$$= \left\{\left(1\frac{1}{3}, 2\frac{2}{3}\right)\right\}$$

New Trend

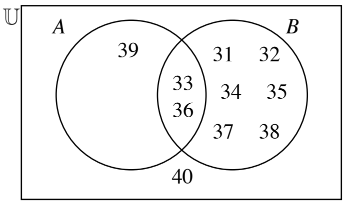
33. $\mathbb{U} = \{x : x$ is an integer, $30 < x \leqslant 40\}$

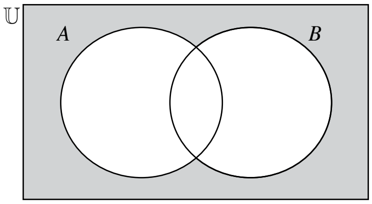
$= \{31, 32, 33, \ldots, 39, 40\}$

$A = \{x : x$ is a multiple of 3$\} = \{33, 36, 39\}$

$B = \{x : 2x - 4 < 73\} = \{31, 32, 33, 34, 35, 36, 37, 38\}$

(a) $A' \cap B = \{31, 32, 34, 35, 37, 38\}$

(b) 

(c) 

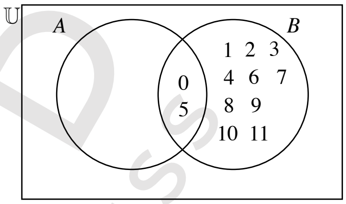
34. $\mathbb{U} = \{x : x$ is an integer, $0 \leqslant x < 12\}$

$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$A = \{x : x(x - 5) = 0\} = \{0, 5\}$

$B = \{x : \frac{1}{3}x - 1 < 3\frac{1}{3}\}$

$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

(a) 

(b) **(i)** $A \cap B = \{0, 5\}$

(ii) $A \cup B' = \{0, 5\}$

35. $\mathbb{U} = \{1, 2, 3, 4, 5, \ldots, 19\}$

(i) $A = \{x : x$ is prime$\} = \{2, 3, 5, 7, 11, 13, 17, 19\}$

(ii) $C = \{x : x$ is a factor of 12$\} = \{1, 2, 3, 4, 6, 12\}$

(iii) $B = \{3, 6, 9, 12, 15, 18\}$

$C' = \{5, 7, 8, 9, 10, 11\}$

$\therefore B \cap C' = \{9\}$

(iv) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 17, 19\}$

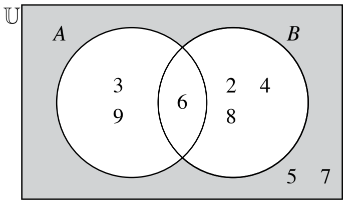
$\therefore (A \cup C)' = \{8, 9, 10, 14, 15, 16, 18\}$

36. $\mathbb{U} = \{x : x$ is a positive integer, $5 < 3x \leqslant 28\}$

$= \{2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{x : x$ is a multiple of 3$\} = \{3, 6, 9\}$

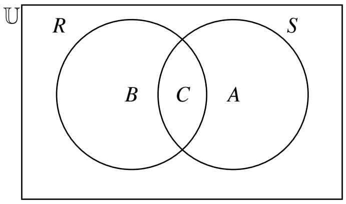
$B = \{x : x$ is divisible by 2$\} = \{2, 4, 6, 8\}$

(i) 

(ii) $(A \cup B)' = \{5, 7\}$

$n(A \cup B)' = 2$

(iii) $A \cap B' = \{3, 9\}$

37. 

Chapter 5 Number Patterns

Basic

1. **(a)** When $n = 1$, $3(1) + 1 = 4$
When $n = 2$, $3(2) + 1 = 7$
When $n = 3$, $3(3) + 1 = 10$
The first three terms are 4, 7 and 10.

(b) When $n = 1$, $2(1) - 7 = -5$
When $n = 2$, $2(2) - 7 = -3$
When $n = 3$, $2(3) - 7 = -1$
The first three terms are -5, -3 and -1.

(c) When $n = 1$, $(1)^2 - 1 = 0$
When $n = 2$, $(2)^2 - 2 = 2$
When $n = 3$, $(3)^2 - 3 = 6$
The first three terms are 0, 2 and 6.

(d) When $n = 1$, $2(1)^2 - 3(1) + 5 = 4$
When $n = 2$, $2(2)^2 - 3(2) + 5 = 7$
When $n = 3$, $2(3)^2 - 3(3) + 5 = 14$
The first three terms are 4, 7 and 14.

(e) When $n = 1$, $\frac{(1)(1-1)}{2} = \frac{(1)(0)}{2} = 0$
When $n = 2$, $\frac{(2)(2-1)}{2} = \frac{(2)(1)}{2} = 1$
When $n = 3$, $\frac{(3)(3-1)}{2} = \frac{(3)(2)}{2} = 3$
The first three terms are 0, 1 and 3.

(f) When $n = 1$, $\frac{2}{1+1} = 1$
When $n = 2$, $\frac{2}{2+1} = \frac{2}{3}$
When $n = 3$, $\frac{2}{3+1} = \frac{2}{4} = \frac{1}{2}$
The first three terms are 1, $\frac{2}{3}$ and $\frac{1}{2}$.

Intermediate

2. **(i)** 6^{th} line: $\frac{1}{6 \times 7} = \frac{1}{6} - \frac{1}{7}$
7^{th} line: $\frac{1}{7 \times 8} = \frac{1}{7} - \frac{1}{8}$

(ii) $272 = p \times q$
Notice that q is 1 more than p.
By trial and error, $16 \times 17 = 272$
$\therefore p = 16$ and $q = 17$

(iii) $\frac{1}{100} - \frac{1}{101} = \frac{1}{100 \times 101} = \frac{1}{10\,100}$

3. **(i)** 7^{th} line: $7^3 - 7 = 336 = (7 - 1) \times 7 \times (7 + 1)$

(ii) 1320 is divisible by 10. Thus the factors of 1320 are 10, 11 and 12.
$1320 = (11 - 1) \times 11 \times (11 + 1)$
$\therefore n = 11$

(iii) $19^3 - 19 = (19 - 1) \times 19 \times (19 + 1) = 6840$

4. **(a)** **(i)** The next four terms are $15 + 6 = 21$, $21 + 7 = 28$, $28 + 8 = 36$ and $36 + 9 = 45$.

(ii) The next four terms are $35 + 21 = 56$, $56 + 28 = 84$, $84 + 36 = 120$, and $120 + 45 = 165$.

(b) **(i)** 9^{th} line: $9^3 - 9 = 720 = 6 \times 120$
10^{th} line: $10^3 - 10 = 990 = 6 \times 165$

(ii) $k = 6$
$p = 6 \times 84 = 504$
Notice that the number of terms follow the number of terms for the sequence 0, 1, 4, 10, …, 84. Since the 8^{th} term is 84, then
$8^3 - 8 = 504 = 6 \times 84$.
$\therefore m = 8$

5. **(a)** 3^{rd} line: $(1 + 2 + 3)^2 = 36 = (1)^3 + (2)^3 + (3)^3$
4^{th} line: $(1 + 2 + 3 + 4)^2$
$= 100 = (1)^3 + (2)^3 + (3)^3 + (4)^3$

(b) **(i)** When $l = 7$, $(1)^3 + (2)^3 + (3)^3 + (4)^3 + (5)^3 + (6)^3 + (7)^3$
$= (1 + 2 + 3 + 4 + 5 + 6 + 7)^2$
$= (28)^2$
$= 784$

(ii) When $1 = 19$,
$(1)^3 + (2)^3 + (3)^3 + (4)^3 + (5)^3 + (6)^3 + \ldots + (19)^3$
$= (1 + 2 + 3 + 4 + 5 + 6 + \ldots + 19)^2$
$= (190)^2$
$= 36\,100$

(c) $(1 + 2 + 3 + \ldots + n)^2 = 2025 = (45)^2$
We observe that
$45 = 40 + 5 = 4 \times 10 + 5$
$(1 + 9) + (2 + 8) + (3 + 7) + (4 + 6) + 5$
$= 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9$
$\therefore n = 9$

(d) $(1)^3 + (2)^3 + (3)^3 + \ldots + (m)^3 = 78^2$
$(1 + 2 + 3 + 4 + 5 + \ldots + m)^3 = 78^2$
$1 + 2 + 3 + 4 + 5 + \ldots + m = 78$
Consider
78
$= 6 \times 13$
$= (1 + 12) + (2 + 11) + (3 + 10) + (4 + 9) + (5 + 8) + (6 + 7)$
$= 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12$
$\therefore m = 12$

6. $1^2 + 1^2 + 2^2 + 3^2 = 3 \times 5$

$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 5 \times 8$

Notice that numbers along this column follow the sequence 1, 1, 2, 3, 5, 8, 13, …

(i) 7^{th} line: $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 + 13^2$
$= 13 \times 21$

(ii) $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + \ldots + l^2 + m^2 = 55 \times n$

Since the left-hand side follows the given sequence, then $l = 34$ and $m = 55$.

The right-hand side of the equation follows the given sequence too.

$\therefore n = 89$

7. **(a)**

110	209	308	407	506	605	704	803	902
121	220	319	418	517	616	715	814	913
132	231	330	429	528	627	726	825	924
143	242	341	440	539	638	737	836	935
154	253	352	451	550	649	748	847	946
165	264	363	462	561	660	759	858	957
176	275	374	473	572	671	770	869	968
187	286	385	484	583	682	781	880	979
198	297	396	495	594	693	792	891	990

(b) These are some of the possible patterns.

For each column, from top cell to bottom cell, add 11 to each term to get the next term.

For each row, from left cell to right cell, add 99 to each term to get the next term.

For each diagonal, from left to right, add 10 to each term to get the next term.

(c) From the table,

$550 \div 11 = 50 = 5^2 + 5^2 + 0^2$

$803 \div 11 = 73 = 8^2 + 0^2 + 3^2$

$\therefore$ The two multiples are 550 and 803.

Advanced

8. **(a)** Since the common difference is 11, $T_n = 11n + ?$.

The term before T_1 is $c = T_0 = 5 - 11 = -6$.

$\therefore$ General term of the sequence, $T_n = 11n - 6$.

$11n - 6 = 100$

$11n = 106$

$n \approx 9.6$

Thus the largest two-digit number occurs when $n = 9$.

When $n = 9$, $11(9) - 6 = 93$.

(b) To find the formula of the general term, consider the following:

8 , 27, 64, 125, …

$2^3, 3^3, 4^3, 5^3, \ldots$

$\therefore$ General term of the sequence $= n^3$,

$n = 2, 3, 4, 5, \ldots.$

$n^3 = 1000$

$n = 10$

Thus the first four-digit number occurs when $n = 10$. When $n = 9$, the largest three-digit number occurs.

When $n = 9$, $9^3 = 729$.

9. **(a)** To find the formula of the general term, consider the following:

4, 9, 16, 25, …

$2^2, 3^2, 4^2, 5^2, \ldots$

$\therefore$ General term of the sequence $= n^2$,

$n = 2, 3, 4, 5, \ldots.$

$n^2 = 100$

$n = 10$

The smallest three-digit number is 100.

(b) To find the formula of the general term, consider the following

$2 = 2 \times 3^0$

$6 = 2 \times 3^1$

$18 = 2 \times 3^2$

$54 = 2 \times 3^3$

$\therefore$ General term $= 2(3)^{n-1}$

Let $2(3)^{n-1} = 1000$

$3^{n-1} = 500$

By trial and error, $3^5 = 243$ and $3^6 = 729$ and so $n - 1 = 6$

So the smallest four-digit number is $2(3)^6 = 1458$.

New Trend

10. (a) Term is obtained by adding the two terms immediately above.

(b) (i) The next two rows are the 6^{th} and 7^{th} rows.

6^{th} row: 1 5 10 10 5 1

7^{th} row: 1 6 15 20 15 6 1

(ii) Sum of the terms in row 1 = 1

Sum of the terms in row 2 = 1 + 1 = 2

Sum of the terms in row 3 = 1 + 2 + 1 = 4

Sum of the terms in row 4 = 1 + 3 + 3 + 1 = 8

Sum of the terms in row 5

= 1 + 4 + 6 + 4 + 1 = 16

Sum of the terms in row 6

= 1 + 5 + 10 + 10 + 5 + 1 = 32

Sum of the terms in row 7

= 1 + 6 + 15 + 20 + 15 + 6 + 1

= 64

Yes, these sums form a pattern equal to 2^{n-1}, where n is the n^{th} row.

(c) (i) Sum of terms in 11^{th} row = 2^{10} = 1024

(ii) Sum of terms in k^{th} row = 2^{k-1}

(d) (i) Number of terms in k^{th} row = k

(ii) The first two terms in the k^{th} row are 1 and $k - 1$.

Chapter 6 Basic Algebra and Algebraic Manipulation

Basic

1. (a) $\frac{1}{3}x + \frac{1}{5}y - \frac{1}{9}x - \frac{1}{15}y$

$= \frac{1}{3}x - \frac{1}{9}x + \frac{1}{5}y - \frac{1}{15}y$

$= \frac{3}{9}x - \frac{1}{9}x + \frac{3}{15}y - \frac{1}{15}y$

$= \frac{2}{9}y + \frac{2}{15}y$

(b) $\frac{3}{4}a - \frac{1}{5}b + 3a - \frac{4}{7}b$

$= \frac{3}{4}a + 3a - \frac{4}{7}b - \frac{1}{5}b$

$= 3\frac{3}{4}a - \frac{20}{35}b - \frac{7}{35}b$

$= 3\frac{3}{4}a - \frac{27}{35}b$

(c) $\frac{5}{6}c + \frac{8}{7}d - \frac{2}{9}c - \frac{5}{3}d$

$= \frac{5}{6}c - \frac{2}{9}c + \frac{8}{7}d - \frac{5}{3}d$

$= \frac{15}{18}c - \frac{4}{18}c + \frac{24}{21}d - \frac{35}{21}d$

$= \frac{11}{18}c - \frac{11}{21}d$

(d) $5f - \frac{5}{7}h + \frac{7}{8}k - \frac{4}{3}f - \frac{4}{5}h + \frac{12}{11}k$

$= 5f - \frac{4}{3}f - \frac{5}{7}h - \frac{4}{5}h + \frac{12}{11}k + \frac{7}{8}k$

$= 3\frac{2}{3}f - \frac{25}{35}h - \frac{28}{35}h + \frac{96}{88}k + \frac{77}{88}k$

$= 3\frac{2}{3}f - 1\frac{18}{35}h + 1\frac{85}{88}k$

2. (a) $5a + 3b - 2c + \left(3\frac{1}{2}a + 2\frac{1}{2}b - 3\frac{1}{2}c\right)$

$= 5a + 3b - 2c + 3\frac{1}{2}a + 2\frac{1}{2}b - 3\frac{1}{2}c$

$= 5a + 3\frac{1}{2}a + 3b + 2\frac{1}{2}b - 2c - 3\frac{1}{2}c$

$= 8\frac{1}{2}a + 5\frac{1}{2}b - 5\frac{1}{2}c$

(b) $\frac{1}{2}[5y - 2(x - 3y)]$

$= \frac{5}{2}y - (x - 3y)$

$= \frac{5}{2}y - x + 3y$

$= \frac{5}{2}y + 3y - x$

$= \frac{11}{2}y - x$

(c) $\frac{3}{4}[8q - 7p - 3(p - 2q)]$

$= \frac{3}{4}[8q - 7p - 3p + 6q)]$

$= \frac{3}{4}[-7p - 3p + 6q + 8q]$

$= \frac{3}{4}[-10p + 14q]$

$= -\frac{30}{4}p + \frac{42}{4}q$

$= \frac{21q - 15p}{2}$

(d) $\frac{3}{10}[3(5a - b) - 7(2a - 5b)]$

$= \frac{3}{10}[15a - 3b - 14a + 35b]$

$= \frac{3}{10}[15a - 14a + 35b - 3b]$

$= \frac{3}{10}[a + 32b]$

$= \frac{3}{10}a + \frac{96}{10}b$

$= \frac{3}{10}(a + 32b)$

3. (a) $\frac{2(5x - 1)}{3} - \frac{x - 3}{5}$

$= \frac{2(5x - 1) \times 5}{3 \times 5} - \frac{(x - 3) \times 3}{5 \times 3}$

$= \frac{50x - 10}{15} - \frac{(3x - 9)}{15}$

$= \frac{50x - 10 - 3x + 9}{15}$

$= \frac{50x - 3x + 9 - 10}{15}$

$= \frac{47x - 1}{15}$

(b) $\frac{x}{2} + \frac{x - 3}{5} - \frac{x - 4}{4}$

$= \frac{5x}{10} + \frac{2(x - 3)}{10} - \frac{x - 4}{4}$

$= \frac{5x + 2x - 6}{10} - \frac{(x - 4)}{4}$

$= \frac{7x - 6}{10} - \frac{(x - 4)}{4}$

$= \frac{(7x - 6) \times 2}{10 \times 2} - \frac{(x - 4) \times 5}{4 \times 5}$

$= \frac{2(7x - 6)}{20} - \frac{5(x - 4)}{20}$

$= \frac{14x - 12 - 5x + 20}{20}$

$= \frac{9x + 8}{20}$

(c) $\frac{x+5}{3} - \frac{2x-7}{6} + \frac{x}{2}$

$= \frac{2(x+5)}{6} - \frac{(2x-7)}{6} + \frac{x}{2}$

$= \frac{2x+10-2x+7}{6} + \frac{x}{2}$

$= \frac{17}{6} + \frac{3x}{6}$

$= \frac{3x+17}{6}$

(d) $\frac{3x-7}{2} - \frac{x+4}{5} - \frac{3}{4}$

$= \frac{5(3x-7)}{10} - \frac{2(x+4)}{10} - \frac{3}{4}$

$= \frac{15x-35-2x-8}{10} - \frac{3}{4}$

$= \frac{15x-2x-8-35}{10} - \frac{3}{4}$

$= \frac{13x-43}{10} - \frac{3}{4}$

$= \frac{2(13x-43)}{20} - \frac{15}{20}$

$= \frac{2(13x-43)-15}{20}$

$= \frac{26x-86-15}{20}$

$= \frac{26x-101}{20}$

(e) $\frac{x+y}{3} - \frac{2}{5} - \frac{3x-2y}{6}$

$= \frac{5(x+y)}{15} - \frac{6}{15} - \frac{3x-2y}{6}$

$= \frac{5x+5y-6}{15} - \frac{3x-2y}{6}$

$= \frac{2(5x+5y-6)}{30} - \frac{5(3x-2y)}{30}$

$= \frac{10x+10y-12-15x+10y}{30}$

$= \frac{10x-15x+10y+10y-12}{30}$

$= \frac{-5x+20y-12}{30}$

Intermediate

4. Perimeter of Figure 1
$= 7y + 3x + 7y + 3x$
$= 7y + 7y + 3x + 3x$
$= 14y + 6x$
$= (6x + 14y)$ cm
Perimeter of Figure 2
$= 5x + (x + 5y) + 7y$
$= 5x + x + 5y + 7y$
$= (6x + 12y)$ cm
$\therefore$ Since $(6x + 14y) > (6x + 12y)$, Figure 1 has a larger perimeter.

5. **(i)** Amount of money spent on the fruits
$= (120h + 180k)$ paisas

(ii) Number of bags in which each bag contains 2 apples and 3 oranges
$= 120 \div 2$
$= 60$
Total amount of money for which he sold all bags of fruits
$= [60(3h + 4k)]$
$= (180h + 240k)$ paisas

(iii) Amount earned from selling the fruits
$= [60(3h + 4k)] - (120h + 180k)$
$= 180h + 240k - 120h - 180k$
$= 180h - 120h + 240k - 180k$
$= (60h + 60k)$ paisas or PKR $(0.6h + 0.6k)$

6. **(i)** Number of 50-paisa coins Sarah has
$= n - x - 3x$
$= n - 4x$

(ii) Since the number of 10-paisa coins is x, then the number of 50-paisa coins is $\frac{1}{4}x$.
Total value of all the coins
$= 10x + 20(3x) + 50\left(\frac{1}{4}x\right)$
$= 10x + 60x + \frac{50}{4}x$
$= 82\frac{1}{2}x$ paisas

(iii) Ratio of number of 20-paisa coins to 50-paisa coins

$= 5 : 3$

5 parts is $3x$.

1 part is $\frac{3x}{5}$.

3 parts is $\frac{3x}{5} \times 3 = \frac{9x}{5}$.

Total value of all the coins

$= 10x + 20(3x) + 50\left(\frac{9x}{5}\right)$

$= 10x + 60x + 90x$

$= 60x$ paisas

7. (a) $\frac{3(x-2)}{3} + \frac{2(x+3)}{4}$

$= \frac{12(x-2)}{12} + \frac{6(x+3)}{12}$

$= \frac{12(x-2) + 6(x+3)}{12}$

$= \frac{12x - 24 + 6x + 18}{12}$

$= \frac{18x - 6}{12}$

$= \frac{3x - 1}{2}$

(b) $\frac{5(3x+1)}{4} - \frac{7(5x-3)}{12}$

$= \frac{15(3x+1)}{12} - \frac{7(5x-3)}{12}$

$= \frac{45x + 15 - 35x + 21}{12}$

$= \frac{45x - 35x + 15 + 21}{12}$

$= \frac{10x + 36}{12}$

$= \frac{5x + 18}{6}$

(c) $1 + \frac{2x+1}{3} + \frac{4(x-3)}{6}$

$= \frac{3}{3} + \frac{2x+1}{3} + \frac{2(x-3)}{3}$

$= \frac{3 + 2x + 1 + 2(x-3)}{3}$

$= \frac{3 + 2x + 1 + 2x - 6}{3}$

$= \frac{2x + 2x + 1 + 3 - 6}{3}$

$= \frac{4x - 2}{3}$

(d) $\frac{3x - 4y}{6} + \frac{x - 2y}{4} - \frac{x + y}{5}$

$= \frac{2(3x - 4y)}{12} + \frac{3(x - 2y)}{12} - \frac{x + y}{5}$

$= \frac{6x - 8y + 3x - 6y}{12} - \frac{x + y}{5}$

$= \frac{5(9x - 14y)}{60} - \frac{12(x + y)}{60}$

$= \frac{5(9x - 14y) - 12(x + y)}{60}$

$= \frac{45x - 70y - 12x - 12y}{60}$

$= \frac{33x - 82y}{60}$

(e) $\frac{2x - 5}{3} - \frac{x + 4}{6} + \frac{3(5 - x)}{9}$

$= \frac{2(2x - 5)}{6} - \frac{x + 4}{6} + \frac{3(5 - x)}{9}$

$= \frac{2(2x - 5) - (x + 4)}{6} + \frac{3(5 - x)}{9}$

$= \frac{4x - 10 - x - 4}{6} + \frac{3(5 - x)}{9}$

$= \frac{4x - x - 10 - 4}{6} + \frac{3(5 - x)}{9}$

$= \frac{3x - 14}{6} + \frac{3(5 - x)}{9}$

$= \frac{3(3x - 14)}{18} + \frac{6(5 - x)}{18}$

$= \frac{3(3x - 14) + 6(5 - x)}{18}$

$= \frac{9x - 42 + 30 - 6x}{18}$

$= \frac{9x - 6x - 42 + 30}{18}$

$= \frac{3x - 12}{18}$

$= \frac{x - 4}{6}$

(f) $\dfrac{4(3x+4)}{10} - \dfrac{x+7}{15} - \dfrac{2x-1}{5}$

$= \dfrac{12(3x+4)}{30} - \dfrac{2(x+7)}{30} - \dfrac{2x-1}{5}$

$= \dfrac{12(3x+4)-2(x+7)}{30} - \dfrac{2x-1}{5}$

$= \dfrac{36x+48-2x-14}{30} - \dfrac{2x-1}{5}$

$= \dfrac{36x-2x+48-14}{30} - \dfrac{6(2x-1)}{30}$

$= \dfrac{34x+34}{30} - \dfrac{6(2x-1)}{30}$

$= \dfrac{34x+34-12x+6}{30}$

$= \dfrac{34x-12x+34+6}{30}$

$= \dfrac{22x+40}{30}$

$= \dfrac{11x+20}{15}$

(g) $-1 - \dfrac{3(x+7)}{7} - \dfrac{4(2x-1)}{5}$

$= \dfrac{-7}{7} - \dfrac{3(x+7)}{7} - \dfrac{4(2x-1)}{5}$

$= \dfrac{-7-3(x+7)}{7} - \dfrac{4(2x-1)}{5}$

$= \dfrac{-7-3x-21}{7} - \dfrac{4(2x-1)}{5}$

$= \dfrac{-3x-28}{7} - \dfrac{4(2x-1)}{5}$

$= \dfrac{5(-3x-28)}{35} - \dfrac{28(2x-1)}{35}$

$= \dfrac{5(-3x-28)-28(2x-1)}{35}$

$= \dfrac{-15x-140-56x+28}{35}$

$= \dfrac{-15x-56x-140+28}{35}$

$= \dfrac{-71x-112}{35}$

(h) $\dfrac{3x-7}{4} - (x-5) - \dfrac{x-1}{3}$

$= \dfrac{3x-7}{4} - \dfrac{x-1}{3} - (x-5)$

$= \dfrac{3(3x-7)}{12} - \dfrac{4(x-1)}{12} - (x-5)$

$= \dfrac{3(3x-7)-4(x-1)}{12} - (x-5)$

$= \dfrac{9x-21-4x+4}{12} - (x-5)$

$= \dfrac{9x-4x-21+4}{12} - (x-5)$

$= \dfrac{5x-17}{12} - \dfrac{12(x-5)}{12}$

$= \dfrac{5x-17-12x+60}{12}$

$= \dfrac{5x-12x-17+60}{12}$

$= \dfrac{-7x+43}{12}$

(i) $\dfrac{2(3x-1)}{5} - (x-3) - \dfrac{2x+1}{3}$

$= \dfrac{2(3x-1)}{5} - \dfrac{2x+1}{3} - (x-3)$

$= \dfrac{6(3x-1)-5(2x+1)}{15} - (x-3)$

$= \dfrac{18x-6-10x-5}{15} - (x-3)$

$= \dfrac{18x-10x-6-5}{15} - (x-3)$

$= \dfrac{8x-11}{15} - \dfrac{15(x-3)}{15}$

$= \dfrac{8x-11-15(x-3)}{15}$

$= \dfrac{8x-11-15x+45}{15}$

$= \dfrac{8x-15x-11+45}{15}$

$= \dfrac{-7x+34}{15}$

Advanced

8. Let Taj's present age be p years.

Then Ahsan's present age is $5p$ years.

In 5 years' time,

Taj is $(p + 5)$ years old and Ahsan is $(5p + 5)$ years old.

$$p + 5 + (5p + 5) = x$$
$$p + 5 + 5p + 5 = x$$
$$p + 5p + 5 + 5 = x$$
$$6p = x - 5 - 5$$
$$6p = x - 10$$
$$p = \frac{x-10}{6}$$

Taj's present age is $\frac{x-10}{6}$ years old.

9. Total age of the girls

$= (n + 5)q$ years

Total age of the group of boys and girls

$= (m + 2 + n + 5)p$

$= (m + n + 7)p$ years

Total age of the boys

$= p(m + n + 7) - q(n + 5)$ years

Average age of the boys

$= \frac{p(m+n+7) - q(n+5)}{(m+2)}$ years old

10. **(a)** $\frac{2(x-3y)}{4} - \frac{4y-x}{12} - \frac{4(x-5y)}{3}$

$= \frac{6(x-3y)}{12} - \frac{4y-x}{12} - \frac{4(x-5y)}{3}$

$= \frac{6(x-3y)-(4y-x)}{12} - \frac{4(x-5y)}{3}$

$= \frac{6x-18y-4y+x}{12} - \frac{4(x-5y)}{3}$

$= \frac{7x-22y}{12} - \frac{16(x-5y)}{12}$

$= \frac{7x-22y-16(x-5y)}{12}$

$= \frac{7x-22y-16x+80y}{12}$

$= \frac{7x-16x-22y+80y}{12}$

$= \frac{-9x+58}{12}$

(b) $\frac{-4(p-3q)}{5} - \left[\frac{2(q-p)}{20} - \frac{3(p-5q)}{4}\right]$

$= \frac{-4(p-3q)}{5} - \left[\frac{2(q-p)}{20} - \frac{15(p-5q)}{20}\right]$

$= \frac{-4(p-3q)}{5} - \left[\frac{2(q-p)-15(p-5q)}{20}\right]$

$= \frac{-4(p-3q)}{5} - \left[\frac{2q-2p-15p+75q}{20}\right]$

$= \frac{-16(p-3q)}{20} - \left[\frac{-17p+77q}{20}\right]$

$= \frac{-16(p-3q)-(-17p+77q)}{20}$

$= \frac{-16p+48q+17p-77q}{20}$

$= \frac{-16p+17p+48q-77q}{20}$

$= \frac{p-29q}{20}$

(c) $-3 + \frac{2(f-3h)}{21} - \frac{5(h-f)}{7} + \frac{2(-2f-3h)}{3}$

$= -3 + \frac{2(f-3h)}{21} - \frac{15(h-f)}{21} + \frac{2(-2f-3h)}{3}$

$= -3 + \frac{2(f-3h)-15(h-f)}{21} + \frac{2(-2f-3h)}{3}$

$= -3 + \frac{2f-6h-15h+15f}{21} + \frac{2(-2f-3h)}{3}$

$= -3 + \frac{17f-21h}{21} + \frac{2(-2f-3h)}{3}$

$= -3 + \frac{17f-21h}{21} + \frac{14(-2f-3h)}{21}$

$= -3 + \frac{17f-21h+14(-2f-3h)}{21}$

$= -3 + \frac{17f-21h-28f-42h}{21}$

$= -3 + \frac{17f-28f-42h-21h}{21}$

$= \frac{-63}{21} + \frac{-11f-63h}{21}$

$= \frac{-11f-63h-63}{21}$

(d) $\frac{x}{5} - \frac{4}{3x}$

$= \frac{3x^2}{15x} - \frac{20}{15x}$

$= \frac{3x^2-20}{15x}$

(e) $\frac{1}{x} + \frac{1}{2x} + \frac{1}{3x}$

$= \frac{6}{6x} + \frac{3}{6x} + \frac{2}{6x}$

$= \frac{11}{6x}$

(f) $\frac{5}{2x} - \frac{3}{3x} + \frac{7}{x}$

$= \frac{15}{6x} - \frac{6}{6x} + \frac{42}{6x}$

$= \frac{15 - 6 + 42}{6x}$

$= \frac{51}{6x}$

$= \frac{17}{2x}$

(g) $\frac{2x - 3}{5y} - \frac{5 - 2x}{10y} + \frac{x}{y}$

$= \frac{2(2x - 3) - (5 - 2x) + 10x}{10y}$

$= \frac{4x - 6 - 5 + 2x + 10x}{10y}$

$= \frac{4x + 2x + 10x - 6 - 5}{10y}$

$= \frac{16x - 11}{10y}$

New Trend

11. $\frac{a}{5} - \frac{2(3a - 5c)}{6}$

$= \frac{a \times 6}{5 \times 6} - \frac{2(3a - 5c) \times 5}{6 \times 5}$

$= \frac{6a}{30} - \frac{10(3a - 5c)}{30}$

$= \frac{6a - 10(3a - 5c)}{30}$

$= \frac{6a - 30a + 50c}{30}$

$= \frac{-24a + 50c}{30}$

$= \frac{2(-12a + 25c)}{30}$

$= \frac{25c - 12a}{15}$

12. (a) $BC = 23x - 2 - (3x - 2) - (5x + 1) - (6x - 7)$

$= 23x - 3x - 5x - 6x - 2 + 2 - 1 + 7$

$= (9x + 6)$ cm

(b) Since $BC = 2AD$,

$9x + 6 = 2(5x + 1)$

$9x + 6 = 10x + 2$

$x = 4$

Perimeter of trapezium $= 23x - 2$

$= 23(4) - 2$

$= 90$ cm

13. $\frac{3x + 4}{10} - \frac{x + 7}{15} - \frac{2x - 1}{5}$

$= \frac{3(3x + 4)}{30} - \frac{2(x + 7)}{30} - \frac{2x - 1}{5}$

$= \frac{9x + 12 - 2x - 14}{30} - \frac{2x - 1}{5}$

$= \frac{9x - 2x + 12 - 14}{30} - \frac{2x - 1}{5}$

$= \frac{7x - 2}{30} - \frac{6(2x - 1)}{30}$

$= \frac{7x - 2 - 6(2x - 1)}{30}$

$= \frac{7x - 2 - 12x + 6}{30}$

$= \frac{7x - 12x - 2 + 6}{30}$

$= \frac{-5x + 4}{30}$

14. x paisa $\rightarrow$ 1 gram

PKR $y = (100 \times y)$ paisa

$= 100y$ paisa

100 y paisa $\rightarrow \frac{1}{x} \times 100y$

$= \frac{100y}{x}$ grams

Chapter 7 Expansion and Factorisation of Quadratic Expressions

Basic

1. (a) $5a^2 + 2a - 3a^2 - a$
$= 2a^2 + a$

(b) $b^2 - 3b + 4 - 2b^2 + 3b - 7$
$= -b^2 - 3$

(c) $c^2 + 4c + 3 + (-2c^2) + (-c) - 2$
$= c^2 + 4c + 3 - 2c^2 - c - 2$
$= -c^2 + 3c + 1$

(d) $4d^2 - d - 5 - (-2d^2) - (-d) + 6$
$= 4d^2 - d - 5 + 2d^2 + d + 6$
$= 6d^2 + 1$

(e) $8e^2 + 8e + 9 - (5e^2 + 2e - 3)$
$= 8e^2 + 8e + 9 - 5e^2 - 2e + 3$
$= 3e^2 + 6e + 12$

(f) $6f^2 - 4f - 1 - (2f^2 - 7f)$
$= 6f^2 - 4f - 1 - 2f^2 + 7f$
$= 4f^2 + 3f - 1$

(g) $-(2 + g - g^2) + (6g - g^2)$
$= -2 - g + g^2 + 6g - g^2$
$= 5g - 2$

(h) $-(1 + 5h - 3h^2) - (2h^2 + 4h - 7)$
$= -1 - 5h + 3h^2 - 2h^2 - 4h + 7$
$= h^2 - 9h + 6$

2. (a) $8 \times 2h$
$= 16h$

(b) $3h \times 4h$
$= 12h^2$

(c) $(-5h) \times 6h$
$= -30h^2$

(d) $(-10h) \times (-7h)$
$= 70h^2$

3. (a) $5(2a + 3)$
$= 10a + 15$

(b) $-4(5b + 1)$
$= -20b - 4$

(c) $8(c^2 + 2c - 3)$
$= 8c^2 + 16c - 24$

(d) $-2(4 - 6d^2)$
$= -8 + 12d^2$
$= 12d^2 - 8$

(e) $3e(8e + 7)$
$= 24e^2 + 21e$

(f) $-f(9 - f)$
$= -9f + f^2$
$= f^2 - 9f$

(g) $-6g(5g - 1)$
$= -30g^2 + 6g$
$= 6g - 30g^2$

(h) $-2h(-3h - 4)$
$= 6h^2 + 8h$

4. (a) $3(a + 2) + 4(2a + 3)$
$= 3a + 6 + 8a + 12$
$= 11a + 18$

(b) $11(5b - 7) + 9(2 - 3b)$
$= 55b - 77 + 18 - 27b$
$= 28b - 59$

(c) $8(5c - 4) + 3(2 - 4c)$
$= 40c - 32 + 6 - 12c$
$= 28c - 26$

(d) $2d(3d + 4) + d(5d - 2)$
$= 6d^2 + 8d + 5d^2 - 2d$
$= 11d^2 + 6d$

(e) $e(6e - 1) + 2e(e - 2)$
$= 6e^2 - e + 2e^2 - 4e$
$= 8e^2 - 5e$

(f) $4f(1 - 2f) + f(3 - f)$
$= 4f - 8f^2 + 3f - f^2$
$= 7f - 9f^2$

5. (a) $(x + 5)(x + 7)$
$= x^2 + 7x + 5x + 35$
$= x^2 + 12x + 35$

(b) $(2x + 1)(x + 3)$
$= 2x^2 + 6x + x + 3$
$= 2x^2 + 7x + 3$

(c) $(x + 6)(3x + 4)$
$= 3x^2 + 4x + 18x + 24$
$= 3x^2 + 22x + 24$

(d) $(4x + 3)(5x + 6)$
$= 20x^2 + 24x + 15x + 18$
$= 20x^2 + 39x + 18$

6. (a) $a^2 + 20a + 75$

$\times$	a	15
a	a^2	$15a$
5	$5a$	75

$\therefore a^2 + 20a + 75 = (a + 15)(a + 5)$

(b) $b^2 + 19b + 18$

$\times$	b	18
b	b^2	$18b$
1	b	18

$\therefore b^2 + 19b + 18 = (b + 18)(b + 1)$

(c) $c^2 - 11c + 28$

$\times$	c	-7
c	c^2	$-7c$
-4	$-4c$	28

$\therefore c^2 - 11c + 28 = (c - 7)(c - 4)$

(d) $d^2 - 21d + 68$

$\times$	d	-17
d	d^2	$-17d$
-4	$-4d$	68

$\therefore d^2 - 21d + 68 = (d - 17)(d - 4)$

(e) $e^2 + 4e - 77$

$\times$	e	11
e	e^2	$11e$
-7	$-7e$	-77

$\therefore e^2 + 4e - 77 = (e + 11)(e - 7)$

(f) $f^2 + 3f - 154$

$\times$	f	14
f	f^2	$14f$
-11	$-11f$	-154

$\therefore f^2 + 3f - 154 = (f + 14)(f - 11)$

(g) $g^2 - 2g - 35$

$\times$	g	-7
g	g^2	$-7g$
5	$5g$	-35

$\therefore g^2 - 2g - 35 = (g - 7)(g + 5)$

(h) $h^2 - 10h - 171$

$\times$	h	-19
h	h^2	$-19h$
9	$9h$	-171

$\therefore h^2 - 10h - 171 = (h - 19)(h + 9)$

7. (a) $6a^2 + 31a + 5$

$\times$	$6a$	1
a	$6a^2$	a
5	$30a$	5

$\therefore 6a^2 + 31a + 5 = (6a + 1)(a + 5)$

(b) $8b^2 + 30b + 27$

$\times$	$4b$	9
$2b$	$8b^2$	$18b$
3	$12b$	27

$\therefore 8b^2 + 30b + 27 = (4b + 9)(2b + 3)$

(c) $4c^2 - 25c + 6$

$\times$	$4c$	-1
c	$4c^2$	$-c$
-6	$-24c$	6

$\therefore 4c^2 - 25c + 6 = (4c - 1)(c - 6)$

(d) $9d^2 - 36d + 32$

$\times$	$3d$	-8
$3d$	$9d^2$	$-24d$
-4	$-12d$	32

$\therefore 9d^2 - 36d + 32 = (3d - 8)(3d - 4)$

(e) $15e^2 + 2e - 1$

$\times$	$5e$	-1
$3e$	$15e^2$	$-3e$
1	$5e$	-1

$\therefore 15e^2 + 2e - 1 = (5e - 1)(3e + 1)$

(f) $2g^2 - 5g - 3$

$\times$	$2g$	1
g	$2g^2$	g
-3	$-6g$	-3

$\therefore 2g^2 - 5g - 3 = (2g + 1)(g - 3)$

(g) $12h^2 - 31h - 15$

$\times$	$12h$	5
h	$12h^2$	$5h$
-3	$-36h$	-15

$\therefore 12h^2 - 31h - 15 = (12h + 5)(h - 3)$

8. (a) $7a \times 3b$
$= 21ab$

(b) $5c \times (-4d)$
$= -20cd$

(c) $(-10e) \times (-2f)$
$= 20ef$

(d) $\frac{1}{6}g \times 24h$
$= 4gh$

9. (a) $5a(2a + 3b)$
$= 10a^2 + 15ab$

(b) $8c(5c - 2d)$
$= 40c^2 - 16cd$

(c) $9e(-4e + 7f)$
$= -36e^2 + 63ef$

(d) $4h(-2g - 3h)$
$= -8gh - 12h^2$

(e) $-6j(k - 4j)$
$= -6jk + 24j^2$

(f) $-4m(2n + 5m)$
$= -8mn - 20m^2$

(g) $-7p(-3p + 4q)$
$= 21p^2 - 28pq$

(h) $-3r(-2r - s)$
$= 6r^2 + 3rs$

(i) $2u(5u + v - w)$
$= 10u^2 + 2uv - 2uw$

(j) $-6x(3x - 2y + z)$
$= -18x^2 + 12xy - 6xz$

10. (a) $4a(3a - b) + 2a(a - 5b)$
$= 12a^2 - 4ab + 2a^2 - 10ab$
$= 14a^2 - 14ab$

(b) $2c(4d - 3c) + 5c(5c - 2d)$
$= 8cd - 6c^2 + 25c^2 - 10cd$
$= 19c^2 - 2cd$

(c) $3f(2e - 7f) + 2e(6f - 5e)$
$= 6ef - 21f^2 + 12ef - 10e^2$
$= -10e^2 + 18ef - 21f^2$

(d) $5h(-2h - 3g) + 2h(-h + 3g)$
$= -10h^2 - 15gh - 2h^2 + 6gh$
$= -12h^2 - 9gh$

11. (a) $(x + y)(x + 4y)$
$= x^2 + 4xy + xy + 4y^2$
$= x^2 + 5xy + 4y^2$

(b) $(2x + y)(3x + y)$
$= 6x^2 + 2xy + 3xy + y^2$
$= 6x^2 + 5xy + y^2$

(c) $(x^2 + 1)(x + 1)$
$= x^3 + x^2 + x + 1$

(d) $(4x^2 + 3)(2x + 3)$
$= 8x^3 + 12x^2 + 6x + 9$

12. (a) $abc - a^2bc^3$
$= abc(1 - ac^2)$

(b) $2a^2b^3c - 8ab^2c^3$
$= 2ab^2c(ab - 4c^2)$

(c) $6k^2 + 8k^3 - 10k^5$
$= 2k^2(3 + 4k - 5k^3)$

(d) $m^2n - mn^2 + m^2n^2$
$= mn(m - n + mn)$

(e) $p^2q - 2pq^2 + 4p^2q^2$
$= pq(p - 2q + 4pq)$

(f) $2s - 4s^2 + 8st^2$
$= 2s(1 - 2s + 4t^2)$

(g) $12x^3 - 9x^2y^2 + 6xy^3$
$= 3x(4x^2 - 3xy^2 + 2y^3)$

(h) $5y^2z - 3y^3z^2 + 6y^2z^2$
$= y^2z(5 - 3yz + 6z)$

13. (a) $4a(x + y) + 7(x + y)$
$= (4a + 7)(x + y)$

(b) $5b(6x + y) - c(y + 6x)$
$= (5b - c)(6x + y)$

(c) $8d(x - 3y) - e(3y - x)$
$= 8d(x - 3y) + e(x - 3y)$
$= (8d + e)(x - 3y)$

(d) $(x + 5)(x - 1) + a(x + 5)$
$= (x - 1 + a)(x + 5)$

Intermediate

14. (a) $6(a + 3) - 5(a - 4)$
$= 6a + 18 - 5a + 20$
$= a + 38$

(b) $13(5b + 7) - 6(3b - 5)$
$= 65b + 91 - 18b + 30$
$= 47b + 121$

(c) $9(3c - 2) - 5(2 + c)$
$= 27c - 18 - 10 - 5c$
$= 22c - 28$

(d) $8(5 - 4d) - 7(7 - 5d)$
$= 40 - 32d - 49 + 35d$
$= 3d - 9$

(e) $7(12 - 5e) - 3(9 - 7e)$
$= 84 - 35e - 27 + 21e$
$= 57 - 14e$

(f) $5f(f + 3) - 4f(5 - f)$
$= 5f^2 + 15f - 20f + 4f^2$
$= 9f^2 - 5f$

(g) $-2g(4 - g) - 3g(2g + 1)$
$= -8g + 2g^2 - 6g^2 - 3g$
$= -4g^2 - 11g$

(h) $-5h(3h + 7) - 4h(-h - 2)$
$= -15h^2 - 35h + 4h^2 + 8h$
$= -11h^2 - 27h$

15. (a) $(y + 7)(y - 11)$
$= y^2 - 11y + 7y - 77$
$= y^2 - 4y - 77$

(b) $(y - 6)(y + 8)$
$= y^2 + 8y - 6y - 48$
$= y^2 + 2y - 48$

(c) $(y - 9)(y - 4)$
$= y^2 - 4y - 9y + 36$
$= y^2 - 13y + 36$

(d) $(2y + 3)(4y - 5)$
$= 8y^2 - 10y + 12y - 15$
$= 8y^2 + 2y - 15$

(e) $(5y - 9)(6y - 1)$
$= 30y^2 - 5y - 54y + 9$
$= 30y^2 - 59y + 9$

(f) $(4y - 1)(3 - 4y)$
$= 12y - 16y^2 - 3 + 4y$
$= -16y^2 + 16y - 3$

(g) $(7 - 2y)(4 + y)$
$= 28 + 7y - 8y - 2y^2$
$= 28 - y - 2y^2$

(h) $(7 - 3y)(8 - 5y)$
$= 56 - 35y - 24y + 15y^2$
$= 56 - 59y + 15y^2$

16. (a) $4 + (a + 2)(a + 5)$
$= 4 + a^2 + 5a + 2a + 10$
$= a^2 + 7a + 14$

(b) $6b + (3b + 1)(b - 2)$
$= 6b + 3b^2 - 6b + b - 2$
$= 3b^2 + b - 2$

(c) $(7c + 2)(3c - 8) + 9c(c + 1)$
$= 21c^2 - 56c + 6c - 16 + 9c^2 + 9c$
$= 30c^2 - 41c - 16$

(d) $(4d - 5)(8d - 7) + (2d + 3)(d - 3)$
$= 32d^2 - 28d - 40d + 35 + 2d^2 - 6d + 3d - 9$
$= 34d^2 - 71d + 26$

17. (a) $-x^2 - 4x + 21$

×	$-x$	3
x	$-x^2$	$3x$
7	$-7x$	21

$\therefore -x^2 - 4x + 21 = (-x + 3)(x + 7)$

(b) $-6x^2 + 2x + 20 = -2(3x^2 - x - 10)$

×	$3x$	5
x	$3x^2$	$5x$
-2	$-6x$	-10

$\therefore -6x^2 + 2x + 20 = -2(3x + 5)(x - 2)$

(c) $12hx^2 - 25hx + 12h = h(12x^2 - 25x + 12)$

×	$4x$	-3
$3x$	$12x^2$	$-9x$
-4	$-16x$	12

$\therefore 12hx^2 - 25hx + 12h = h(4x - 3)(3x - 4)$

18. $3x^2 + 26x + 51$

×	$3x$	17
x	$3x^2$	$17x$
3	$9x$	51

$\therefore 3x^2 + 26x + 51 = (3x + 17)(x + 3)$
$32\ 651 = 3(100)^2 + 26(100) + 51$
Let $x = 100$.
$32\ 651 = 317 \times 103$
$\therefore$ The factors are 317 and 103.

19. $4x^2 + 13x + 3$

×	$4x$	1
x	$4x^2$	x
3	$12x$	3

$\therefore 4x^2 + 13x + 3 = (4x + 1)(x + 3)$
$41\ 303 = 4(100)^2 + 13(100) + 3$
Let $x = 100$.
$41\ 303 = 401 \times 103$
$\therefore$ The prime factors are 401 and 103.

20. (a) $\frac{1}{2}a \times \frac{2}{3}b$
$= \frac{1}{3}ab$

(b) $\frac{2}{5}c \times \left(-\frac{3}{8}d\right)$
$= -\frac{3}{20}cd$

(c) $\left(-\frac{1}{4}e\right) \times \frac{12}{13}f$
$= -\frac{3}{13}ef$

(d) $\left(-\frac{6}{7}g\right) \times \left(-\frac{7}{12}h\right)$
$= \frac{1}{2}gh$

(e) $0.2p \times 12q$
$= 2.4pq$

(f) $3r \times 0.9s$
$= 2.7rs$

(g) $4w^2x \times 5wx^3$
$= 20w^3x^4$

(h) $(-8xy^2z) \times (-2xz^3)$
$= 16x^2y^2z^4$

21. $\frac{4}{5}a^2bc^3 \times \frac{15}{16}ab^2$
$= \frac{3}{4}a^3b^3c^3$

22. **(a)** $5ab(a - 4b)$
$= 5a^2b - 20ab^2$

(b) $-3c(2c^2d + d^2)$
$= -6c^3d - 3cd^2$

(c) $-8ef(6f - e^2)$
$= -48ef^2 + 8e^3f$

(d) $-10h^2(-7g^2h - 9h^3)$
$= 70g^2h^3 + 90h^5$

23. **(a)** $4a(3b + 5c) - 3b(8c - 9a)$
$= 12ab + 20ac - 24bc + 27ab$
$= 39ab + 20ac - 24bc$

(b) $5d(2d + 5e) - 3e(2e - 7d)$
$= 10d^2 + 25de - 6e^2 + 21de$
$= 10d^2 + 46de - 6e^2$

(c) $7f(2f + 3g) - 3f(-4g + 3f)$
$= 14f^2 + 21fg + 12fg - 9f^2$
$= 5f^2 + 33fg$

(d) $4h(-3h + k) - 2h(-5k + h)$
$= -12h^2 + 4hk + 10hk - 2h^2$
$= -14h^2 + 14hk$

24. **(a)** $(a + 6b)(a - 2b)$
$= a^2 - 2ab + 6ab - 12b^2$
$= a^2 + 4ab - 12b^2$

(b) $(4c + 5d)(5c + 7d)$
$= 20c^2 + 28cd + 25cd + 35d^2$
$= 20c^2 + 53cd + 35d^2$

(c) $(4e - 3f)(2e + 7f)$
$= 8e^2 + 28ef - 6ef - 21f^2$
$= 8e^2 + 22ef - 21f^2$

(d) $(2g - 3h)(g - 2h)$
$= 2g^2 - 4gh - 3gh + 6h^2$
$= 2g^2 - 7gh + 6h^2$

(e) $(m^2 - 4)(2m + 3)$
$= 2m^3 + 3m^2 - 8m - 12$

(f) $(2n - 4)(n^2 + 3)$
$= 2n^3 + 6n - 4n^2 - 12$
$= 2n^3 - 4n^2 + 6n - 12$

(g) $(2p - 3q)(2p - 5r)$
$= 4p^2 - 10pr - 6pq + 15qr$

(h) $(xy - 5)(xy + 8)$
$= x^2y^2 + 8xy - 5xy - 40$
$= x^2y^2 + 3xy - 40$

25. **(a)** $(a + 1)(a - 3) + (2a - 3)(5 - 7a)$
$= a^2 - 3a + a - 3 + 10a - 14a^2 - 15 + 21a$
$= -13a^2 + 29a - 18$

(b) $(7b + 1)(b - 5) - 3(4 - 2b - b^2)$
$= 7b^2 - 35b + b - 5 - 12 + 6b + 3b^2$
$= 10b^2 - 28b - 17$

(c) $(3c - 8)(c + 1) - (2c - 1)(5 - c)$
$= 3c^2 + 3c - 8c - 8 - (10c - 2c^2 - 5 + c)$
$= 3c^2 - 5c - 8 - (-2c^2 + 11c - 5)$
$= 3c^2 - 5c - 8 + 2c^2 - 11c + 5$
$= 5c^2 - 16c - 3$

(d) $(d + 3e)(d - 3e) - 2(d + 2e)(d - e)$
$= d^2 - 9e^2 - 2(d^2 - de + 2de - 2e^2)$
$= d^2 - 9e^2 - 2(d^2 + de - 2e^2)$
$= d^2 - 9e^2 - 2d^2 - 2de + 4e^2$
$= -d^2 - 2de - 5e^2$

26. **(a)** $a^2 + 7ab + 6b^2$

$\times$	a	b
a	a^2	ab
$6b$	$6ab$	$6b^2$

$\therefore a^2 + 7ab + 6b^2 = (a + b)(a + 6b)$

(b) $c^2 + 11cd - 12d^2$

$\times$	c	$12d$
c	c^2	$12cd$
$-d$	$-cd$	$-12d^2$

$\therefore c^2 + 11cd - 12d^2 = (c + 12d)(c - d)$

(c) $2d^2 - de - 15e^2$

$\times$	$2d$	$5e$
d	$2d^2$	$5de$
$-3e$	$-6de$	$-15e^2$

$\therefore 2d^2 - de - 15e^2 = (2d + 5e)(d - 3e)$

(d) $6f^2 - 29fg + 28g^2$

$\times$	$3f$	$-4g$
$2f$	$6f^2$	$-8fg$
$-7g$	$-21fg$	$28g^2$

$\therefore 6f^2 - 29fg + 28g^2 = (3f - 4g)(2f - 7g)$

(e) $2m^2 + 2mn - 12n^2 = 2(m^2 + mn - 6n^2)$

×	m	$-2n$
m	m^2	$-2mn$
$3n$	$3mn$	$-6n^2$

$\therefore\ 2m^2 + 2mn - 12n^2 = 2(m - 2n)(m + 3n)$

(f) $px^2 - 11pxy + 24py^2 = p(x^2 - 11xy + 24y^2)$

×	x	$-3y$
x	x^2	$-3xy$
$-8y$	$-8xy$	$24y^2$

$\therefore\ px^2 - 11pxy + 24py^2 = p(x - 3y)(x - 8y)$

27. $12x^2 + xy - 20y^2$

×	$4x$	$-5y$
$3x$	$12x^2$	$-15xy$
$4y$	$16xy$	$-20y^2$

$12x^2 + xy - 20y^2 = (4x - 5y)(3x + 4y)$

$\therefore$ Breadth of rectangle $= \dfrac{(4x - 5y)(3x + 4y)}{4x - 5y}$

$= (3x + 4y)$ cm

28. (a) $(2a + b)(x + y) + (a + b)(x + y)$
$= (2a + b + a + b)(x + y)$
$= (3a + 2b)(x + y)$

(b) $(4c + 3d)^2 + (4c + 3d)(c + d)$
$= (4c + 3d)(4c + 3d + c + d)$
$= (4c + 3d)(5c + 4d)$

(c) $2p(5r - 7s) + 3q(7s - 5r)$
$= 2p(5r - 7s) - 3q(5r - 7s)$
$= (2p - 3q)(5r - 7s)$

(d) $9w(y - x) - 8z(x - y)$
$= 9w(y - x) + 8z(y - x)$
$= (9w + 8z)(y - x)$

29. (a) $p^2 + pq + 3qr + 3pr$
$= p(p + q) + 3r(q + p)$
$= (p + 3r)(p + q)$

(b) $3xy + 6y - 5x - 10$
$= 3y(x + 2) - 5(x + 2)$
$= (3y - 5)(x + 2)$

(c) $x^2z - 4y - x^2y + 4z$
$= x^2z - x^2y + 4z - 4y$
$= x^2(z - y) + 4(z - y)$
$= (x^2 + 4)(z - y)$

(d) $x^3 + xy - 3x^2y - 3y^2$
$= x(x^2 + y) - 3y(x^2 + y)$
$= (x^2 + y)(x - 3y)$

(e) $x - 4x^2 - 4 + x^3$
$= x^3 + x - 4x^2 - 4$
$= x(x^2 + 1) - 4(x^2 + 1)$
$= (x^2 + 1)(x - 4)$

(f) $h^2 - 1 + hk + k$
$= (h + 1)(h - 1) + k(h + 1)$
$= (h - 1 + k)(h + 1)$

(g) $m - n - m^2 + n^2$
$= (m - n) - (m^2 - n^2)$
$= (m - n) - (m + n)(m - n)$
$= (1 - m - n)(m - n)$

(h) $a^2 - 3bc - ab + 3ac$
$= a^2 - ab + 3ac - 3bc$
$= a(a - b) + 3c(a - b)$
$= (a + 3c)(a - b)$

(i) $x^2y - 3y - 6 + 2x^2$
$= y(x^2 - 3) - 2(3 - x^2)$
$= y(x^2 - 3) + 2(x^2 - 3)$
$= (y + 2)(x^2 - 3)$

(j) $a^2x - 12by - 3bx + 4a^2y$
$= a^2x + 4a^2y - 3bx - 12by$
$= a^2(x + 4y) - 3b(x + 4y)$
$= (x + 4y)(a^2 - 3b)$

Advanced

30. (a) $9a^2 - (4a - 1)(a + 2)$
$= 9a^2 - (4a^2 + 8a - a - 2)$
$= 9a^2 - (4a^2 + 7a - 2)$
$= 9a^2 - 4a^2 - 7a + 2$
$= 5a^2 - 7a + 2$

(b) $3b(2 - b) - (1 + b)(1 - b)$
$= 6b - 3b^2 - (1 - b + b - b^2)$
$= 6b - 3b^2 - (1 - b^2)$
$= 6b - 3b^2 - 1 + b^2$
$= -2b^2 + 6b - 1$

(c) $(5c + 6)(6c - 5) - (3 - 2c)(1 - 15c)$
$= 30c^2 - 25c + 36c - 30 - (3 - 45c - 2c + 30c^2)$
$= 30c^2 + 11c - 30 - (3 - 47c + 30c^2)$
$= 30c^2 + 11c - 30 - 3 + 47c - 30c^2$
$= 58c - 33$

(d) $(2d - 8)\left(\frac{1}{2}d - 4\right) - (3d - 6)\left(\frac{1}{3}d + 1\right)$
$= d^2 - 8d - 4d + 32 - (d^2 + 3d - 2d - 6)$
$= d^2 - 12d + 32 - (d^2 + d - 6)$
$= d^2 - 12d + 32 - d^2 - d + 6$
$= 38 - 13d$

31. (i) $3x^2 + 48x + 189 = 3(x^2 + 16x + 63)$

×	x	7
x	x^2	$7x$
9	$9x$	63

$\therefore\ 3x^2 + 48x + 189 = 3(x + 7)(x + 9)$

(ii) $969 = 3(10)^2 + 48(10) + 189$

Let $x = 10$.

$969 = 3 \times 17 \times 19$

$\therefore$ Sum $= 3 + 17 + 19$
$= 39$

32. $2x^2 - 2.9x - 3.6 = 0.1(20x^2 - 29x - 36)$

×	$5x$	4
$4x$	$20x^2$	$16x$
-9	$-45x$	-36

$\therefore\ 2x^2 - 2.9x - 3.6 = 0.1(5x + 4)(4x - 9)$

i.e. $p = 5,\ q = 4,\ r = 4,\ s = -9$

$\therefore\ p + q + r + s = 5 + 4 + 4 - 9$
$= 4$

33. (a) $(2h + 3)(h - 7) - (h + 4)(h^2 - 1)$
$= 2h^2 - 14h + 3h - 21 - (h^3 - h + 4h^2 - 4)$
$= 2h^2 - 11h - 21 - h^3 + h - 4h^2 + 4$
$= -h^3 - 2h^2 - 10h - 17$

(b) $(3p^2 + q)(2p - q) - (2p + q)(3p^2 - q)$
$= 6p^3 - 3p^2q + 2pq - q^2 - (6p^3 - 2pq + 3p^2q - q^2)$
$= 6p^3 - 3p^2q + 2pq - q^2 - 6p^3 + 2pq - 3p^2q + q^2$
$= 4pq - 6p^2q$

34. (a) $(2a + 1)(a^2 - 3a - 4)$
$= 2a^3 - 6a^2 - 8a + a^2 - 3a - 4$
$= 2a^3 - 5a^2 - 11a - 4$

(b) $(b + 2)(3b^2 - 5b + 6)$
$= 3b^3 - 5b^2 + 6b + 6b^2 - 10b + 12$
$= 3b^3 + b^2 - 4b + 12$

(c) $(7 - c)(5c^2 - 2c + 1)$
$= 35c^2 - 14c + 7 - 5c^3 + 2c^2 - c$
$= -5c^3 + 37c^2 - 15c + 7$

(d) $(d^2 - 4)(d^2 - 2d + 1)$
$= d^4 - 2d^3 + d^2 - 4d^2 + 8d - 4$
$= d^4 - 2d^3 - 3d^2 + 8d - 4$

(e) $(h - 2k)(2h + 3k - 1)$
$= 2h^2 + 3hk - h - 4hk - 6k^2 + 2k$
$= 2h^2 - hk - 6k^2 - h + 2k$

(f) $(m - n)(m^2 + mn + n^2)$
$= m^3 + m^2n + mn^2 - m^2n - mn^2 - n^3$
$= m^3 - n^3$

(g) $(p + 1)(p^3 - p^2 + p - 1)$
$= p^4 - p^3 + p^2 - p + p^3 - p^2 + p - 1$
$= p^4 - 1$

(h) $(q - 1)(q^3 - 3q^2 + 3q - 1)$
$= q^4 - 3q^3 + 3q^2 - q - q^3 + 3q^2 - 3q + 1$
$= q^4 - 4q^3 + 6q^2 - 4q + 1$

New Trend

35. (a) $16a^2 - 9b^2$

×	$4a$	$3b$
$14a$	$16a^2$	$12ab$
$-3b$	$-12ab$	$-9b^2$

$\therefore\ 16a^2 - 9b^2 = (4a + 3b)(4a - 3b)$

(b) $3f^2 + 11f - 20$

×	$3f$	-4
f	$3f^2$	$-4f$
5	$15f$	-20

$\therefore\ 3f^2 + 11f - 20 = (3f - 4)(f + 5)$

(c) $9x^2 - 15x - 6 = 3(3x^2 - 5x - 2)$

×	$3x$	1
x	$3x^2$	x
-2	$-6x$	-2

$\therefore\ 9x^2 - 15x - 6 = 3(3x + 1)(x - 2)$

36. (a) $2ax - 4ay + 3bx - 6by$
$= 2a(x - 2y) + 3b(x - 2y)$
$= (2a + 3b)(x - 2y)$

(b) $5ax - 10ay - 3bx + 6by$
$= 5a(x - 2y) - 3b(x - 2y)$
$= (5a - 3b)(x - 2y)$

(c) $8ab - 6bc + 15cd - 20ad$
$= 2b(4a - 3c) + 5d(3c - 4a)$
$= 2b(4a - 3c) - 5d(4a - 3c)$
$= (2b - 5d)(4a - 3c)$

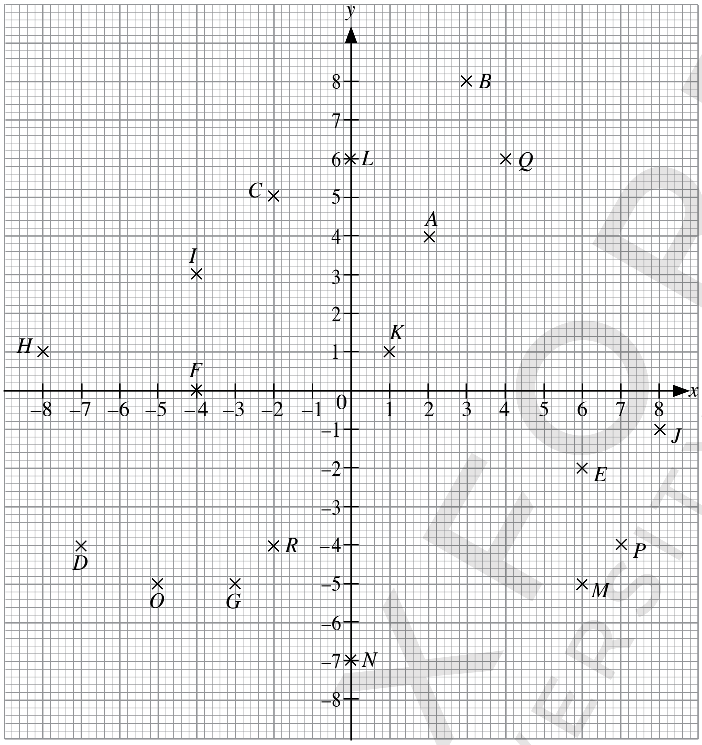
Chapter 8 LInear Equation and Coordinate Geometry

Basic

1. We can find the coordinates from the graph. Each ordered pair determines the points A to R.

$A(1, 2)$	$B(7, 1)$	$C(-2, -3)$
$D(-4, 5)$	$E(6, 6)$	$F(3, -2)$
$G(-6, -2)$	$H(5, 0)$	$I(0, -5)$
$J(-7, 4)$	$L(-3, 0)$	$M(0, 3)$
$N(-5, 2)$	$O(0, 0)$	$P(6, -4)$
$Q(-3, -6)$	$R(4, -6)$	

2.

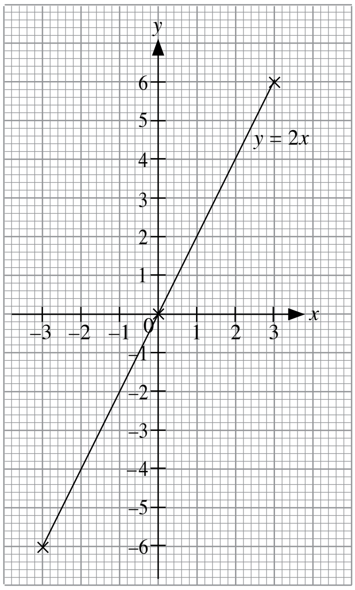


3. **(i)** When $x = 2$, $y = 5 \times 2 + 7 = 17$.
   (ii) When $x = -3$, $y = 5 \times (-3) + 7 = -8$.

4. **(i)** When $y = -8$,
   $-8 = 10 - 9x$
   $9x = 10 + 8 = 18$
   $x = 2$
   (ii) When $y = -26$,
   $-26 = 10 - 9x$
   $9x = 10 + 26 = 36$
   $x = 4$

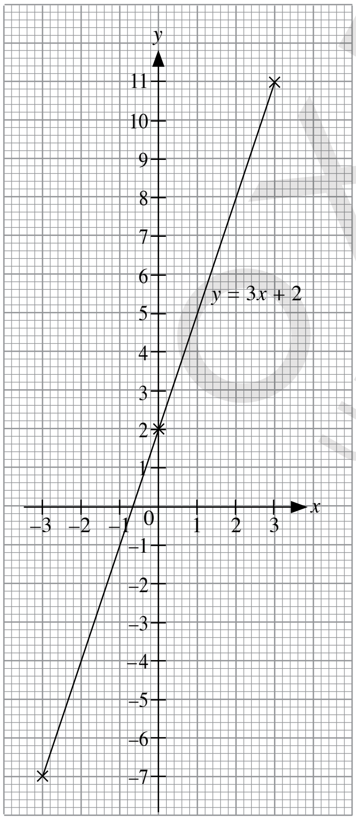
5. **(a)** $y = 2x$

x	-3	0	3
$y = 2x$	-6	0	6



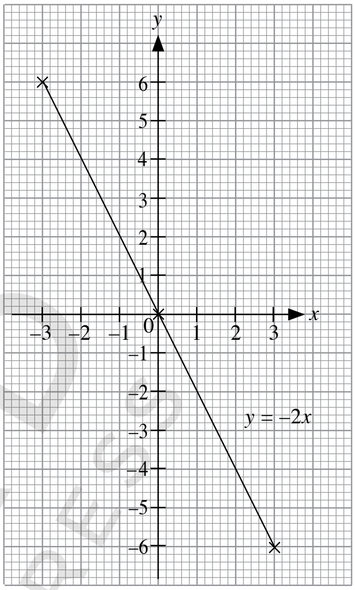
(b) $y = 3x + 2$

x	-3	0	3
$y = 3x + 2$	-7	2	11



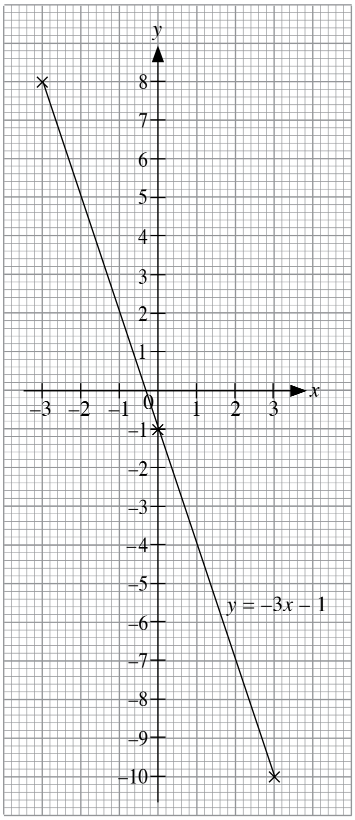
(c) $y = -2x$

x	-3	0	3
$y = -2x$	6	0	-6



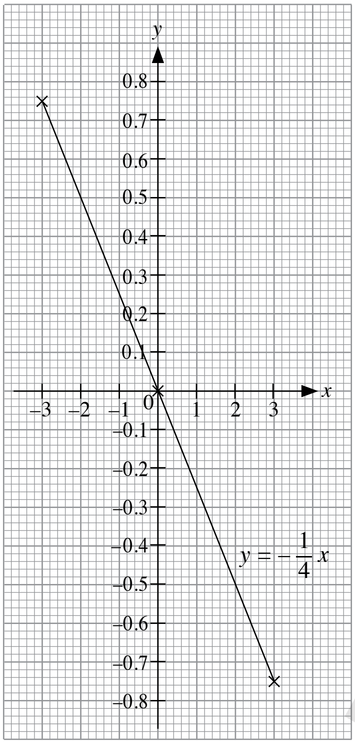
(d) $y = -3x - 1$

x	-3	0	3
$y = -3x - 1$	8	-1	-10



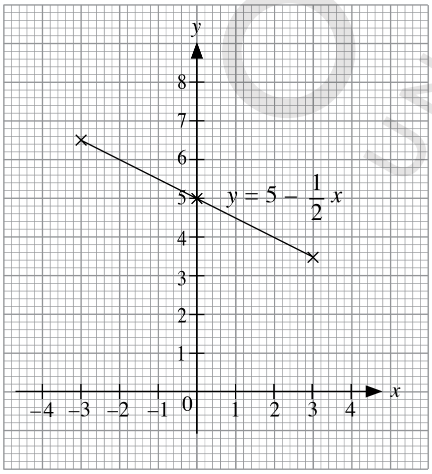
(e) $y = -\frac{1}{4}x$

x	-3	0	3
$y = -\frac{1}{4}x$	$\frac{3}{4}$	0	$-\frac{3}{4}$

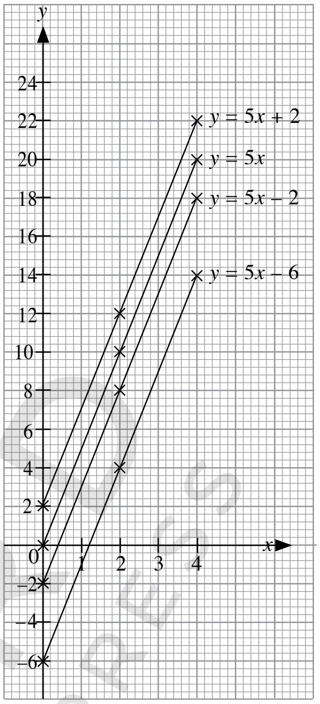


(f) $y = 5 - \frac{1}{2}x$

x	-3	0	3
$y = 5 - \frac{1}{2}x$	$6\frac{1}{2}$	5	$3\frac{1}{2}$

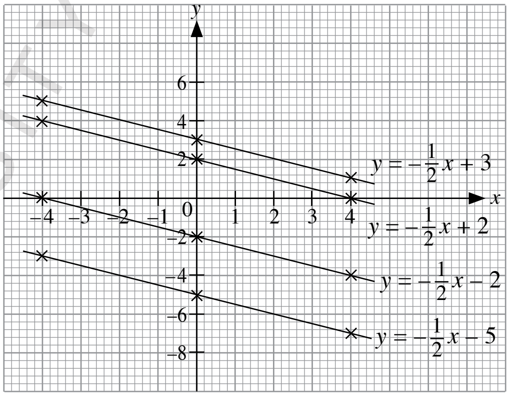


6. **(a)**



(b) They are parallel lines.

7. **(a)**



(b) They are parallel lines.

8. **(a)** **(i)** Amount of money left after 2 days
$= 30 - 5 \times 2 =$ PKR 20

(ii) Amount of money left after 3 days
$= 30 - 5 \times 3 =$ PKR 15

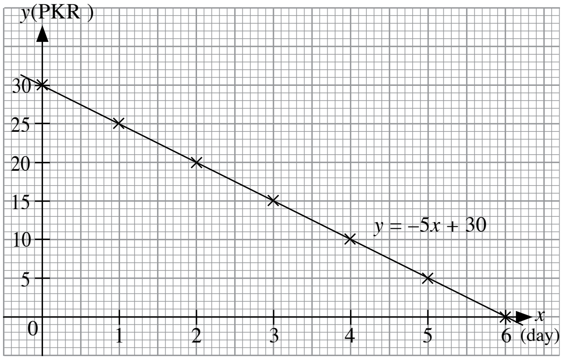
(iii) Amount of money left after 4 days
$= 30 - 5 \times 4 =$ PKR 10

(iv) Amount of money left after 5 days
$= 30 - 5 \times 5 =$ PKR 5

(b)

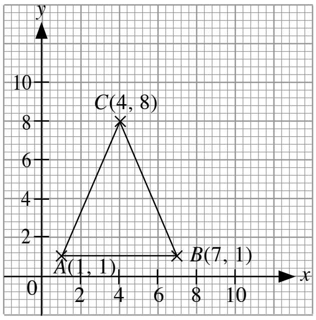
x	0	1	2	3	4	5	6
y	30	25	20	15	10	5	0

(c)



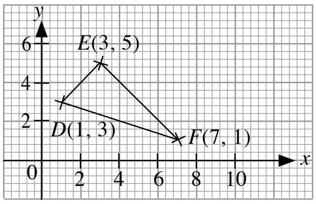
Intermediate

9. (a)



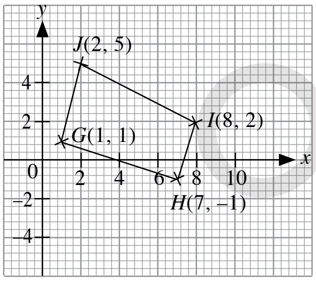
Isosceles triangle

(b)



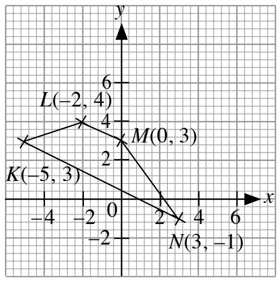
Right-angled triangle

(c)



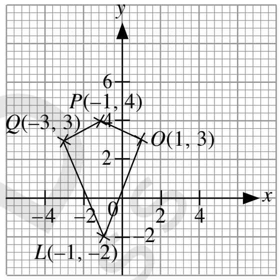
Quadrilateral

(d)



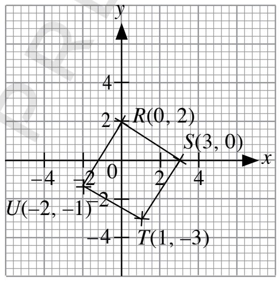
Trapezium

(e)



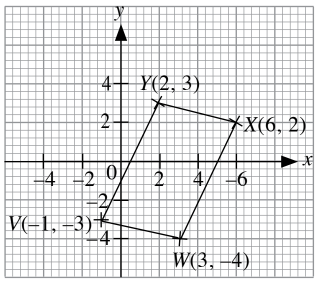
Kite

(f)



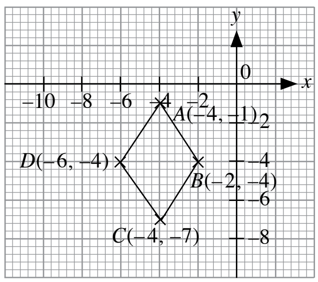
Square

(g)



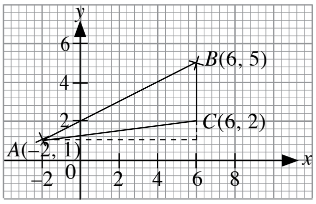
Parallelogram

(h)



Rhombus

10.

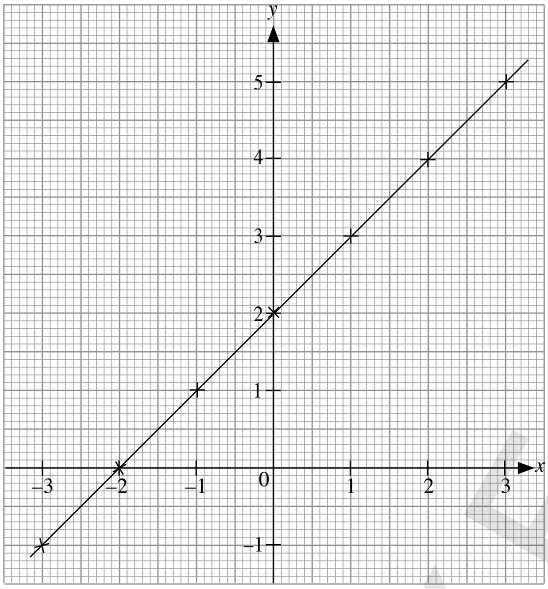


Area of triangle $ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 3 \times 8$

$= 12$ square units

11.



The points can be joined to form a straight line that slopes upwards from left to right.

12. **(i)** When $x = 5$, $y = \frac{3}{5} \times 5 - \frac{1}{2} = 2\frac{1}{2}$.

(ii) When $x = 3\frac{1}{2}$, $y = \frac{3}{5} \times 3\frac{1}{2} - \frac{1}{2} = 1\frac{3}{5}$.

(iii) When $x = -\frac{2}{3}$, $y = \frac{3}{5} \times \left(-\frac{2}{3}\right) - \frac{1}{2} = -\frac{9}{10}$.

13. **(i)** When $y = 12$,

$12 = 15 - \frac{3}{4}x$

$\frac{3}{4}x = 15 - 12$

$\frac{3}{4}x = 3$

$3x = 12$

$x = 4$

(ii) When $y = 21$,

$21 = 15 - \frac{3}{4}x$

$\frac{3}{4}x = 15 - 21$

$\frac{3}{4}x = -6$

$3x = -24$

$x = -8$

(iii) When $y = -60$,

$-60 = 15 - \frac{3}{4}x$

$\frac{3}{4}x = 15 + 60$

$\frac{3}{4}x = 75$

$3x = 300$

$x = 100$

14. For the line $y = 3x + 2$,

(a) When $x = 1$, $y = 5$, then
LHS = 5
RHS = $3 \times 1 + 2 = 5$
Since LHS = RHS, then $A(1, 5)$ lies on the line.

(b) When $x = 3$, $y = 12$, then
LHS = 12
RHS = $3 \times 3 + 2 = 11$
Since LHS ≠ RHS, then $B(3, 12)$ does not lie on the line.

(c) When $x = 0$, $y = 2$, then
LHS = 2
RHS = $3 \times 0 + 2 = 2$
Since LHS = RHS, then $C(0, 2)$ lies on the line.

(d) When $x = -2$, $y = 4$, then
LHS = 4
RHS = $3 \times (-2) + 2 = -4$
Since LHS ≠ RHS, then $D(-2, 4)$ does not lie on the line.

(e) When $x = -\frac{1}{3}$, $y = 1$, then

LHS = 1

RHS = $3 \times \left(-\frac{1}{3}\right) + 2 = 1$

Since LHS = RHS, then $E\left(-\frac{1}{3}, 1\right)$ lies on the line.

15. For the line $y = -\frac{1}{2}x - 2$,

(a) When $x = 2$, $y = -1$, then

LHS $= -1$

RHS $= -\frac{1}{2} \times 2 - 2 = -3$

Since LHS $\neq$ RHS, then $A(2, -1)$ does not lie on the line.

(b) When $x = -4$, $y = 0$, then

LHS $= 0$

RHS $= -\frac{1}{2} \times (-4) - 2 = 0$

Since LHS = RHS, then $B(-4, 0)$ lies on the line.

(c) When $x = \frac{2}{3}$ and $y = -\frac{7}{3}$, then

LHS $= -\frac{7}{3}$

RHS $= -\frac{1}{2} \times \frac{2}{3} - 2 = -\frac{7}{3}$

Since LHS = RHS, then $C\left(\frac{2}{3}, -\frac{7}{3}\right)$ lies on the line.

(d) When $x = -\frac{1}{2}$, $y = -\frac{7}{4}$, then

LHS $= -\frac{7}{4}$

RHS $= -\frac{1}{2} \times \left(-\frac{1}{2}\right) - 2 = -\frac{7}{4}$

Since LHS = RHS, then $D\left(-\frac{1}{2}, -\frac{7}{4}\right)$ lies on the line.

(e) When $x = 10$, $y = -3$, then

LHS $= -3$

RHS $= -\frac{1}{2} \times 10 - 2 = -7$

Since LHS $\neq$ RHS, then $E(10, -3)$ does not lie on the line.

16. (a) When $x = -3$,

$y = \frac{3 \times (-3) + 7}{2} = \frac{-2}{2} = -1$

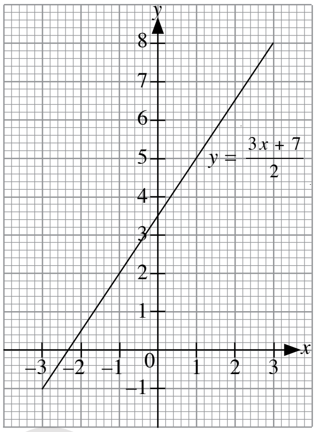
When $x = 0$,

$y = \frac{3 \times 0 + 7}{2} = \frac{7}{2}$

When $x = 3$,

$y = \frac{3 \times 3 + 7}{2} = \frac{16}{2} = 8$

x	-3	0	3
y	-1	$\frac{7}{2}$	8



(b) When $x = -3$,

$y = \frac{-2 \times (-3) + 1}{2} = \frac{7}{2}$

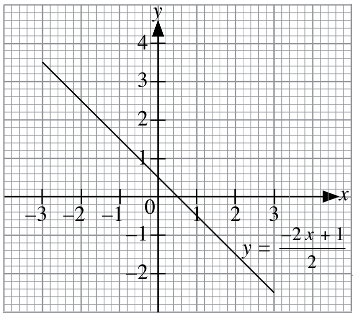
When $x = 0$,

$y = \frac{-2 \times 0 + 1}{2} = \frac{1}{2}$

When $x = 3$,

$y = \frac{-2 \times 3 + 1}{2} = -\frac{5}{2}$

x	-3	0	3
y	$\frac{7}{2}$	$\frac{1}{2}$	$-\frac{5}{2}$



(c) When $x = -3$,

$$2y + 3 \times (-3) = 4$$
$$2y - 9 = 4$$
$$2y = 4 + 9$$
$$2y = 13$$
$$y = \frac{13}{2}$$

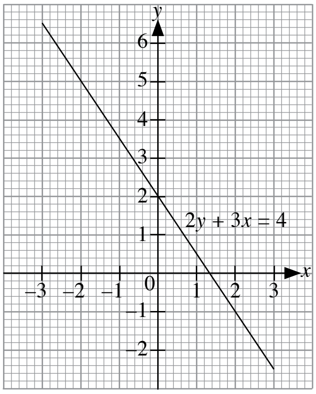
When $x = 0$,

$$2y + 3 \times 0 = 4$$
$$2y = 4$$
$$y = 2$$

When $x = 3$,

$$2y + 3 \times 3 = 4$$
$$2y + 9 = 4$$
$$2y = 4 - 9$$
$$2y = -5$$
$$y = -\frac{5}{2}$$

x	–3	0	3
y	$\frac{13}{2}$	2	$-\frac{5}{2}$



(d) When $x = -3$,

$$5y - 2 \times (-3) = 8$$
$$5y + 6 = 8$$
$$5y = 8 - 6$$
$$5y = 2$$
$$y = \frac{2}{5}$$

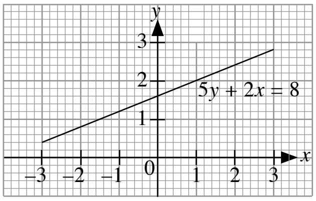
When $x = 0$,

$$5y - 2 \times 0 = 8$$
$$5y = 8$$
$$y = \frac{8}{5}$$

When $x = 3$,

$$5y - 2 \times 3 = 8$$
$$5y - 6 = 8$$
$$5y = 8 + 6$$
$$5y = 14$$
$$y = \frac{14}{5}$$

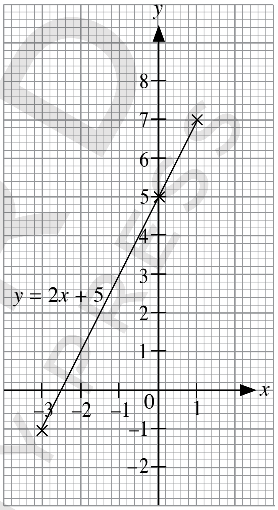
x	–3	0	3
y	$\frac{2}{5}$	$\frac{8}{5}$	$\frac{14}{5}$



17. (a)

x	–3	0	1
$y = 2x + 5$	$y = 2 \times (-3) + 5$ $= -1$	$y = 2 \times 0 + 5$ $= 5$	$y = 2 \times 1 + 5$ $= 7$

(b)

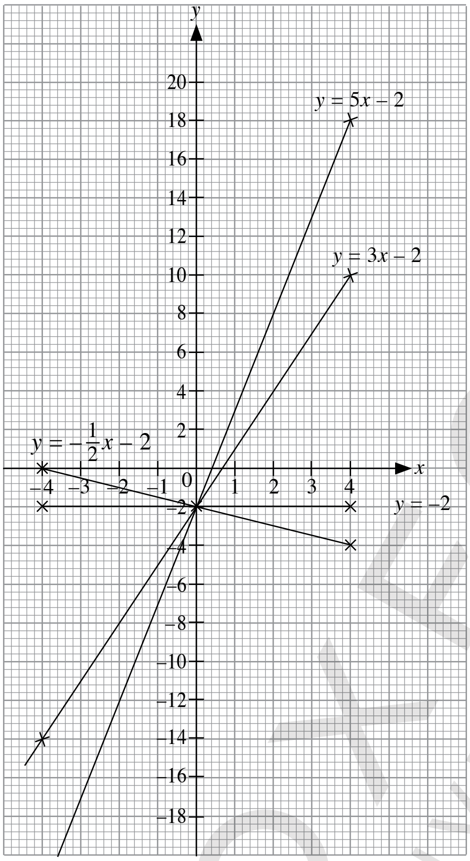


(c) From the graph, the value of x is about $-5\frac{3}{4}$.

(Note: It is necessary to extrapolate the graph so that we can find the value of x when y is less than –6.)

18. (a)

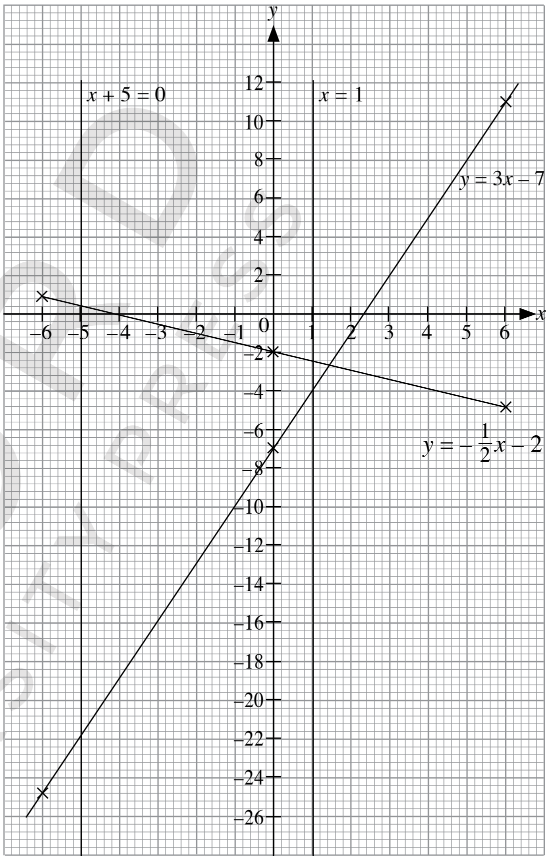
x	-4	0	4
$y = 3x - 2$	$y = 3 \times (-4) - 2$ $= -14$	$y = 3 \times 0 - 2$ $= -2$	$y = 3 \times 4 - 2 = 10$
$y = 5x - 2$	$y = 5 \times (-4) - 2$ $= -22$	$y = 5 \times 0 - 2$ $= -2$	$y = 5 \times 4 - 2 = 18$
$y = -\frac{1}{2}x - 2$	$y = -\frac{1}{2} \times (-4) - 2$ $= 0$	$y = -\frac{1}{2} \times 0 - 2$ $= -2$	$y = -\frac{1}{2} \times 4 - 2$ $= -4$
$y = -2$	$y = -2$	$y = -2$	$y = -2$



(b) All the lines pass through the point (0, –2).

19. (a)

x	-6	0	6
$x = 1$	N.A.	N.A.	N.A.
$x + 5 = 0$	N.A.	N.A.	N.A.
$y = -\frac{1}{2}x - 2$	$y = -\frac{1}{2} \times (-6) - 2$ $= 1$	$y = -\frac{1}{2} \times 0 - 2$ $= -2$	$y = -\frac{1}{2} \times 6 - 2$ $= -5$
$y = 3x - 7$	$y = 3 \times (-6) - 7$ $= -25$	$y = 3 \times 0 - 7$ $= -7$	$y = 3 \times 6 - 7$ $= 11$



(b) The shape of the figure formed by the lines is a trapezium.

(c) In order to find the area bounded by the lines, locate the coordinates of the points of intersection of the lines.

From the graph, the coordinates of points of intersections of the lines are

(1, –2.5), (1, –4), (–5, 0.5) and (–5, –22)

Area bounded by the lines

$= \frac{1}{2} \times (1.5 + 22.5) \times 6$

$= 72$ square units

20. **(a)** **(i)** Amount of money the house owner has to pay
= 35 + 1 × 15 = PKR 50

(ii) Amount of money the house owner has to pay
= 35 + 2 × 15 = PKR 65

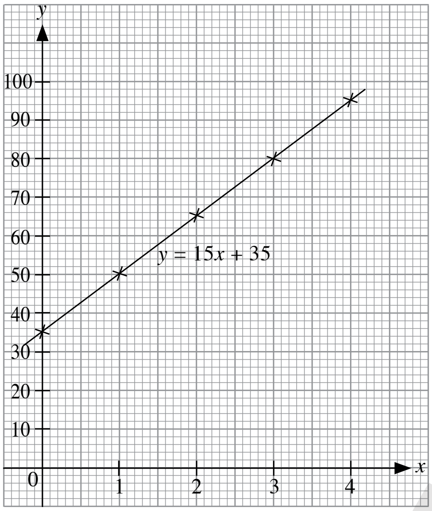
(iii) Amount of money the house owner has to pay
= 35 + 3 × 15 = PKR 80

(iv) Amount of money the house owner has to pay
= 35 + 4 × 15 = PKR 95

(b)

x	0	1	2	3	4
y	35	50	65	80	95

(c)



(d) **(i)** From the graph, the amount of money charged if he spends 2.5 hours on the job
= PKR 72.50

(ii) From the graph, the number of hours that the electrician spends on the job
≈ 3.65 hours

21. **(a)** From the graph, the values of C can be obtained.

N	50	100	150	200
C	150	200	250	300

(b) **(i)** From the graph, the value of m is 100.

(ii) He has to pay PKR m for the operation cost of printing the newsletters.

(c) From the graph, the amount of money Taj has to pay is PKR 175.

(d) From the graph, the maximum number of newsletters he print is 155.

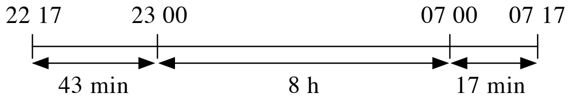
Chapter 9 Time and Speed

Basic

1. 5.55 p.m. + 40 min = 6.35 p.m.

= 18 35

2.



Total time taken = 43 min + 8 h + 17 min

= 9 h

3. **(a)** $84 \text{ km/h} = \dfrac{84 \text{ km}}{1 \text{ h}}$

$= \dfrac{84\ 000 \text{ m}}{3600 \text{ s}}$

$= 23\dfrac{1}{3}$ m/s

(b) $15 \text{ m/s} = \dfrac{15 \text{ m}}{1 \text{ s}}$

$= \dfrac{(15 \div 1000) \text{ km}}{(1 \div 3600) \text{ s}}$

= 54 km/h

(c) $\dfrac{2}{3} \text{ km/min} = \dfrac{\frac{2}{3} \text{ km}}{1 \text{ min}}$

$= \dfrac{\frac{2}{3} \text{ km}}{(1 \div 60) \text{ h}}$

= 40 km/h

(d) $120 \text{ cm/s} = \dfrac{120 \div 100}{1 \text{ s}}$

= 1.2 m/s

4. Convert 44 minutes to hours.

$44 \text{ min} = \dfrac{44}{60} = \dfrac{11}{15}$ h

Time taken to travel a distance of 1 km $= \dfrac{11}{15} \div 11$

$= \dfrac{1}{15}$ h

(a) **(i)** Time taken to travel a distance of 45 km

$= \dfrac{1}{15} \times 45$

= 3 h

(ii) Time taken to travel a distance of 36 km

$= \dfrac{1}{15} \times 36$

$= 2\dfrac{2}{5}$ h or 2 h 24 min

(iii) Time taken to travel a distance of 20 km

$= \dfrac{1}{15} \times 20$

$= 1\dfrac{1}{3}$ h or 1 h 20 min

(b) Speed of the cyclist

$= \dfrac{11 \times 1000}{44 \times 60}$

$= 4\dfrac{1}{6}$ m/s

5. **(i)** Time taken for the journey

= 50 min + 3 h 24 min + 2 h 6 min + 1 h 30 min

$= \dfrac{5}{6} \text{ h} + 3\dfrac{2}{5} \text{ h} + 2\dfrac{1}{10} \text{ h} + 1\dfrac{1}{2} \text{ h}$

$= 7\dfrac{5}{6}$ h or 7 h 50 min

(ii) Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{687}{7\frac{5}{6}}$

= 87.7 km/h (to 3 s.f.)

6. **(i)** Convert 36 minutes to hours.

$36 \text{ min} = \dfrac{36}{60} = \dfrac{3}{5}$ h

Average speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \dfrac{27}{\frac{3}{5}}$

= 45 km/h

(ii) Time at which Maaz reaches the station

= 0837 + 36 min

= 0913

Time at which the train arrives at the station

= 0942 + 11 min

= 0953

Waiting time = 0953 – 0913 = 40 min

7. **(i)** Time taken by the car for the whole journey

= 1510 – 0845

= 6 h 25 min

$= 6\dfrac{5}{12}$ h

(ii) Distance = speed × time

$= 84 \times 6\dfrac{5}{12}$

= 539 km

8. **(i)** Convert 54 minutes to hours.

$54 \text{ min} = \frac{54}{60} = \frac{9}{10} \text{ h}$

Distance travelled for the first part of the journey

$= 70 \times \frac{9}{10}$

$= 63 \text{ km}$

Distance travelled for the return journey

$= 63 \text{ km}$

Time taken for the return journey

$= \frac{63}{45}$

$= 1\frac{2}{5} \text{ h or } 1 \text{ h } 24 \text{ min}$

(ii) Time at which Hussain starts to return to the original point

= 0955 + 54 min + 40 min

= 1129

Time when Hussain arrives at the starting point

= 1129 + 1 h 24 min

= 1253

Intermediate

9. **(i)** Time for which the car is parked

= 1630 – 0745

$= 8 \text{ h } 45 \text{ min or } 8\frac{3}{4} \text{ h}$

(ii) Parking fee

= PKR 2.50 + 14 × PKR 0.80 + PKR 0.80 + PKR 0.80

= PKR 15.30

10. Distance travelled by the wheel = 765 × 2.8

= 2142 m

Number of revolutions made by the wheel to travel a distance of 2142 m

$= \frac{2142}{1.7}$

= 1260 times

11. Convert 46 minutes to hours.

$46 \text{ min} = \frac{46}{60} = \frac{23}{30} \text{ h}$

Let the time taken to travel from Town Y to Z be T hours.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$80 = \frac{80 + 48}{T + \frac{23}{30}}$$

$$80\left(T + \frac{23}{30}\right) = 128$$

$$80T + 61\frac{1}{3} = 128$$

$$80T = 128 - 61\frac{1}{3}$$

$$= 66\frac{2}{3}$$

$$T = \frac{5}{6} \text{ h}$$

Speed of the driver when he is driving from Town Y to Z

$$= \frac{80}{\frac{5}{6}}$$

= 96 km/h

12. **(i)** Time arrived at B = 1035 + 0019

= 1054

Time arrived at C = 1150 + (0019 – 0011)

= 1158

(ii) Time to travel from Town C to D

= 1320 – 1158

= 1 h 22 min

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{123}{1\frac{22}{60}}$$

= 90 km/h

Advanced

13. $\frac{a - 2b}{10} = \frac{b}{6}$

$$6(a - 2b) = 10b$$

$$6a - 12b = 10b$$

$$6a = 10b + 12b$$

$$6a = 22b$$

$$\frac{6a}{b} = 22$$

$$\frac{a}{b} = \frac{22}{6} = \frac{11}{3}$$

The ratio of $a : b = 11 : 3$.

14. Let the distance travelled by the motorist be y km.

$y = x \times 2\frac{1}{2}$

$= 2\frac{1}{2}x$ — (1)

$y = (x + 4) \times \left(2\frac{1}{2} - \frac{15}{60}\right)$

$= 2\frac{1}{4}(x + 4)$ — (2)

Substitute (1) into (2):

$$2\frac{1}{2}x = 2\frac{1}{4}(x + 4)$$

$$2\frac{1}{2}x = 2\frac{1}{4}x + 9$$

$$2\frac{1}{2}x - 2\frac{1}{4}x = 9$$

$$\frac{1}{4}x = 9$$

$$x = 36$$

The value of x is 36.

15. Time taken for the van to travel a distance of 130 km

$= \frac{130}{65}$

$= 2$ h

Time taken for the car to travel a distance of 130 km

$= 2 - \frac{35}{60}$

$= 1\frac{5}{12}$ h

Average speed $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{130}{1\frac{5}{12}}$

$= 91\frac{13}{17}$ km/h

New Trend

16. (a) 280 km/h $= \frac{280\text{ km}}{1\text{ h}}$

$= \frac{280\ 000\text{ m}}{3600\text{ s}}$

$= 77\frac{7}{9}$ m/s

(b) Time take for bullet train to pass through tunnel completely

$= \frac{(20\ 500 + 250)\text{ m}}{77\frac{7}{9}\text{ m/s}}$

$= 266\frac{11}{14}$ s

$= 4$ min 27 s (to the nearest second)

17. Time taken to fly from Place A to Place B

$= \frac{9257}{752}$

$= 12.3125$ h

$= 12$ h 0.1325×60 min

$= 12$ h 19 min (to the nearest minute)

18. (i) Distance travelled on 1 litre of petrol

$= \frac{128}{12}$

$= 10\frac{2}{3}$ km

Distance travelled on 30 litres of petrol

$= 10\frac{2}{3} \times 30$

$= 320$ km

(ii) Amount of petrol required to travel a distance of 1 km

$= \frac{12}{128}$ litres

Amount of petrol required to travel a distance of 15 000 km

$= \frac{12}{128} \times 15\ 000$

$= 1406.25$ litres

Amount the car owner has to pay

$= 1406.25 \times$ PKR 2.03

$=$ PKR 2854.69 (to the nearest paisa)

19. (a) 180 km $\rightarrow$ 50.4 litres

100 km $\rightarrow \frac{50.4}{180} \times 100$

$= 28$ litres

The fuel consumption of the bus is 28 l/100 km.

(b) (i) 7.6 litres $\rightarrow$ 100 km

50 litres $\rightarrow \frac{100}{7.6} \times 50$

$= 658$ km (to 3 s.f.)

(ii) 100 km $\rightarrow$ 7.6 litres

330 km $\rightarrow \frac{7.6}{100} \times 330$

$= 25.08$ litres

1 litre $\rightarrow$ PKR 2.07

25.08 litres $\rightarrow$ PKR 2.07×25.08

$=$ PKR 51.92 (to the nearest paisa)

The petrol will cost Ali PKR 51.92 for a journey of 330 km.

Chapter 10 Triangles, Quadrilaterals and Polygons

Basic

1. **(a)** $2x° + 46° + 82° = 180°$ ($\angle$ sum of $\triangle$)
$2x° = 180° - 46° - 82°$
$= 52°$
$x° = 26°$
$\therefore x = 26$

(b) $x° + 58° + 58° = 180°$ ($\angle$ sum of $\triangle$)
$x° = 180° - 58° - 58°$
$= 64°$
$\therefore x = 64$

(c) $x° + x° + 70° = 180°$
$2x° = 180° - 70°$
$= 110°$
$x° = 55°$
$\therefore x = 55$

(d) $3x° = 63°$
$x° = 21°$
$\therefore x = 21$

(e) $3y° = 48°$ (base $\angle$s of isos. $\triangle$)
$y° = 16°$
$2x° + 3y° + 48° = 180°$ ($\angle$ sum of $\triangle$)
$2x° = 180° - 3y° - 48°$
$= 180° - 3(16°) - 48°$
$= 180° - 48° - 48°$
$= 84°$
$x° = 42°$
$\therefore x = 42$ and $y = 16$

2. **(a)** $x° + 39° = 123°$ (ext. $\angle$ of $\triangle$)
$x° = 123° - 39°$
$= 84°$
$\therefore x = 84$

(b) $y° + 40° = 180°$ (adj. $\angle$s on a str. line)
$y° = 180° - 40°$
$y° = 140°$
$4x° + 3x° = y° = 140°$ (ext. $\angle$ of $\triangle$)
$7x° = 140°$
$x° = 20°$
$\therefore x = 20$ and $y = 140$

(c) $26° + 26° = x°$ (ext. $\angle$ of $\triangle$)
$x° = 52°$
$\therefore x = 52$

(d) $\angle CAD = 180° - 110°$ (adj. $\angle$s on a str. line)
$= 70°$
$\angle CDA = \angle CAD = 70°$ (base $\angle$s of isos. $\triangle ACD$)
$x° + 70° = 110°$ (ext. $\angle$ of $\triangle$)
$x° = 110° - 70°$
$= 40°$
$\therefore x = 40$

(e) $\angle EAD = x°$ (vert. opp. $\angle$s)
$x° + 72° + 50° = 180°$ ($\angle$ sum of $\triangle$)
$x° = 180° - 72° - 50°$
$= 58°$
$\therefore x = 58$

(f) $\angle DAC = 60°$ ($\angle$s of equilateral $\triangle ACD$)
$2y° + 2y° = 60°$ (base $\angle$s of isos. $\triangle ACB$,
$4y° = 60°$ ext. $\angle$ of $\triangle$)
$y° = 15°$
$\therefore y = 15$

3. **(a)** $\angle CAB = 46°$ (alt. $\angle$s, DE // AB)
$x° + 46° = 91°$ (ext. $\angle$ of $\triangle ACB$)
$x° = 91° - 46°$
$= 45°$
$\therefore x = 45$

(b) $\angle CBA = 3x°$ (alt. $\angle$s, CD // AB)
$3x° + 2x° + 55° = 180°$ ($\angle$ sum of $\triangle ACB$)
$5x° = 180° - 55°$
$= 125°$
$x° = 25°$
$\therefore x = 25$

(c) $\angle BDA = x°$ (base $\angle$s of isos. $\triangle ABD$)
$x° + x° + x° + 63° = 180°$ ($\angle$ sum of $\triangle ADC$)
$3x° = 180° - 63° = 117°$
$x° = 39°$
$\therefore x = 39$

(d) $\angle DBA = 58°$ (alt. $\angle$s, DE // AB)
$x° + 58° = 79°$ (ext. $\angle$ of $\triangle ACB$)
$x° = 79° - 58°$
$= 21°$
$y° + 79° + 3x° = 180°$ ($\angle$ sum of $\triangle ACD$)
$3x° + y° = 180° - 79° = 101°$
$y° = 101° - 3x°$
$= 101° - 3(21°)$
$= 38°$
$\therefore x = 21$ and $y = 38$

4. **(a)** $2x° + 62° = 134°$ (ext. ∠ of △)
$2x° = 134° - 62°$
$= 72°$
$x° = 36°$
$\angle BCE = 2x°$ (alt. ∠s, CE // AB)
$y° + 134° + 2x° = 360°$ (∠s at a point)
$y° = 360° - 134° - 2(36°)$
$= 360° - 134° - 72°$
$= 154°$
$\therefore x = 36$ and $y = 154$

(b) $\angle ACD = 180° - 109°$ (int. ∠s, ED // AF)
$= 71°$
$x° + 24° = 71°$ (ext. ∠ of △ABC)
$x° = 71° - 24°$
$= 47°$
$\therefore x = 47$

(c) $y° + 63° = 142°$ (ext. ∠ of △)
$y° = 142° - 63°$
$= 79°$
$\angle ADF + 63° = 180°$ (int. ∠s, EF // AC)
$\angle ADF = 180° - 63°$
$= 117°$
$x° = \angle ADF = 117°$ (vert. opp. ∠s)
$\therefore x = 117$ and $y = 79$

(d) $\angle DEC = y°$ (alt. ∠s, ED // AC)
$4x° + y° = 180°$ (adj. ∠s on a str. line)
$y° = 180° - 4x°$
$\angle ECD = 180° - y° - 36°$
$= 144° - y°$
$144° - y° + 2x° = 4x°$ (ext. ∠ of △DEC)
$144° - (180° - 4x°) + 2x° = 4x°$
$2x° = 36°$
$x° = 18°$
$y° = 180° - 4(18°)$
$= 108°$
$\therefore x = 18$ and $y = 108$

5. **(i)** $\angle BAC = 36°$ (base ∠s of isos. △ABC)
$\angle ACD = \angle ABC + \angle BAC$ (ext. ∠ of △ABC)
$= 36° + 36°$
$= 72°$
$\angle ADC = \angle ACD = 72°$ (base ∠s of isos. △ACD)
$\angle CAD = 180° - 72° - 72°$ (∠ sum of △ACD)
$= 36°$

(ii) $\angle ADE = \angle CAD + \angle ACD$ (ext. ∠ of △ACD)
$= 72° + 36°$
$= 108°$

6. **(a)** $x° + 29° = 90°$ (∠DAB is a right angle)
$x° = 90° - 29°$
$= 61°$
$y° = \angle BAC = 29°$ (alt. ∠s, DC // AB)
$\therefore x = 61$ and $y = 29$

(b) $x° = \dfrac{180° - 118°}{2}$ (base ∠s of isos. △)
$= 31°$
$\angle CBD + 31° = 90°$ (∠CBA is a right angle)
$\angle CBD = 90° - 31°$
$= 59°$
$y° + 59° = 118°$ (ext. ∠ of △)
$y° = 118° - 59°$
$= 59°$
$\therefore x = 31$ and $y = 59$

(c) $x° + x° = (3x - 18)°$ (ext. ∠ of △)
$2x° = 3x° - 18°$
$x° = 18°$
$y° + x° + 90° - x° + x° = 180°$ (∠ sum of △ABD)
$y° = 180° - 90° - x°$
$= 180° - 90° - 18°$
$= 72°$
$\therefore x = 18$ and $y = 72$

(d) $2x° + 2x° = 180° - (162 - 3x)°$ (ext. ∠ of △)
$4x° = 180° - 162° + 3x°$
$x° = 18°$
$\angle DAC = 90° - 2(18°)$
$= 54°$
$y° + 54° = (162 - 3x)°$ (ext. ∠ of △)
$y° = (162 - 3x)° - 54°$
$= (162 - 3(18))° - 54°$
$= 108° - 54°$
$= 54°$
$\therefore x = 18$ and $y = 54$

7. **(a)** $y° = 120°$ (opp. ∠s of //gram)
$x° + 24° + y° = 180°$ (∠ sum of △ABC)
$x° = 180° - 24° - y°$
$= 180° - 24° - 120°$
$= 36°$
$\therefore x = 36$ and $y = 120$

(b) $7x° + 5x° = 180°$ (int. ∠s, DC // AB)
$12x° = 180°$
$x° = 15°$
$2y° = 5x°$ (opp. ∠s of //gram)
$2y° = 5(15°)$
$2y° = 75°$
$y° = 37.5°$
$\therefore x = 15$ and $y = 37.5$

(c) $\angle DAB = 180° - 68°$ (int. $\angle$s, AD // BC)
$= 112°$
$x° + 112° = 139°$ (ext. $\angle$ of $\triangle$)
$x° = 139° - 112°$
$= 27°$
$y° + x° = 68°$ (opp. $\angle$s of //gram)
$y° = 68° - x°$
$= 68° - 27°$
$= 41°$
$\therefore x = 27$ and $y = 41$

8. **(a)** $y° = \dfrac{180° - 58°}{2}$ (base $\angle$s of isos. $\triangle ABC$)
$= 61°$
$x° = 180° - 31° - 31°$ (base $\angle$s of isos. $\triangle ACD$)
$= 118°$
$\therefore x = 118$ and $y = 61$

(b) $x° + 33° + 56° = 180°$ ($\angle$ sum of $\triangle ABD$)
$x° = 180° - 33° - 56°$
$= 91°$
$y° = 33°$
$\therefore x = 91$ and $y = 33$

(c) $(x + 5)° + 90° + 28° = 180°$ ($\angle$ sum of $\triangle$)
$x° + 5° + 90° + 28° = 180°$
$x° = 180° - 5° - 90° - 28°$
$= 57°$
$(y - 6)° + 47° + 90° = 180°$ ($\angle$ sum of $\triangle$)
$y° - 6° + 47° + 90° = 180°$
$y° = 180° + 6° - 47° - 90°$
$= 49°$
$\therefore x = 57$ and $y = 49$

(d) $(x - 5)° + 63° + 90° = 180°$ ($\angle$ sum of $\triangle$)
$x° - 5° + 63° + 90° = 180°$
$x° = 180° + 5° - 63° - 90°$
$= 32°$
$(2y - 3)° + 37° + 90° = 180°$ ($\angle$ sum of $\triangle$)
$2y° - 3° + 37° + 90° = 180°$
$2y° = 180° + 3° - 37° - 90°$
$= 56°$
$y° = 28°$
$\therefore x = 32$ and $y = 28$

9. **(a)** Sum of interior angles of a polygon with 7 sides
$= (n - 2) \times 180°$
$= (7 - 2) \times 180°$
$= 900°$

(b) Sum of interior angles of a polygon with 17 sides
$= (n - 2) \times 180°$
$= (17 - 2) \times 180°$
$= 2700°$

(c) Sum of interior angles of a polygon with 22 sides
$= (n - 2) \times 180°$
$= (22 - 2) \times 180°$
$= 3600°$

(d) Sum of interior angles of a polygon with 30 sides
$= (n - 2) \times 180°$
$= (30 - 2) \times 180°$
$= 5040°$

10. **(a)** Sum of interior angles of a polygon with 4 sides
$= (n - 2) \times 180°$
$= (4 - 2) \times 180°$
$= 360°$
$a° + 125° + 65° + 92° = 360°$
$a° = 360° - 125° - 65° - 92°$
$= 78°$
$\therefore a = 78$

(b) Sum of interior angles of a polygon with 4 sides
$= (n - 2) \times 180°$
$= (4 - 2) \times 180°$
$= 360°$
$2b° + 105° + 75° + b° = 360°$
$2b° + b° = 360° - 105° - 75°$
$3b° = 180°$
$b° = 60°$
$\therefore b = 60$

(c) Sum of interior angles of a polygon with 5 sides
$= (n - 2) \times 180°$
$= (5 - 2) \times 180°$
$= 540°$
$c° + (2c - 15)° + 130° + 65° + 120° = 540°$
$c° + 2c° = 540° + 15° - 130° - 65° - 120°$
$3c° = 240°$
$c° = 80°$
$\therefore c = 80$

11. Let the number of sides of the regular polygon be n.
Size of each interior angle $= \dfrac{(n - 2) \times 180°}{n}$

(a) $\dfrac{(n - 2) \times 180°}{n} = 108°$
$(n - 2) \times 180 = 108n$
$180n - 108n = 2 \times 180$
$72n = 360$
$n = 5$

(b) $\dfrac{(n - 2) \times 180°}{n} = 156°$
$(n - 2) \times 180 = 156n$
$180n - 156n = 2 \times 180$
$24n = 360$
$n = 15$

12. Let the number of sides of the regular polygon be n.

Size of each exterior angle $= \frac{360°}{n}$.

(a) $\frac{360°}{n} = 5°$

$5n = 360$

$n = 72$

(b) $\frac{360°}{n} = 6°$

$6n = 360$

$n = 60$

(c) $\frac{360°}{n} = 8°$

$8n = 360$

$n = 45$

(d) $\frac{360°}{n} = 18°$

$18n = 360$

$n = 20$

13. (a) Size of each exterior angle $= \frac{360°}{6} = 60°$

(b) Size of each exterior angle $= \frac{360°}{8} = 45°$

(c) Size of each exterior angle $= \frac{360°}{24} = 15°$

(d) Size of each exterior angle $= \frac{360°}{72} = 5°$

14. (a) The sum of interior angles of the polygon is 1620°.

i.e. $(n - 2) \times 180° = 1620°$

$\therefore$ Number of sides of the polygon

$= \frac{1620}{180} + 2 = 11$

(b) The sum of interior angles of the polygon is 3600°.

i.e. $(n - 2) \times 180° = 3600°$

$\therefore$ Number of sides of the polygon

$= \frac{3600}{180} + 2 = 22$

(c) The sum of interior angles of the polygon is 4500°.

i.e. $(n - 2) \times 180° = 4500°$

$\therefore$ Number of sides of the polygon

$= \frac{4500}{180} + 2 = 27$

(d) The sum of interior angles of the polygon is 7020°.

i.e. $(n - 2) \times 180° = 7020°$

$\therefore$ Number of sides of the polygon

$= \frac{7020}{180} + 2 = 41$

15. (i) The sum of exterior angles of a triangle = 360°.

$(2x + 10)° + (3x - 5)° + (2x + 40)° = 360°$

$2x° + 3x° + 2x° = 360° - 10° + 5° - 40°$

$7x° = 315°$

$x° = 45°$

$\therefore x = 45$

(ii) The largest exterior angle gives the smallest interior angle.

The largest exterior angle

$= (3x - 5)°$ or $(2x + 40)°$

$= (3 \times 45 - 5)°$ or $(2 \times 45 + 40)°$

$= 130°$

The smallest interior angle = 180° – 130° = 50°

(iii) The smallest exterior angle gives the largest interior angle.

The smallest exterior angle $= (2x + 10)°$

$= (2 \times 45 + 10)°$

$= 100°$

The largest interior angle = 180° – 100° = 80°

16. (i) Sum of interior angles of a quadrilateral

$= (n - 2) \times 180°$

$= (4 - 2) \times 180°$

$= 360°$

$(2x + 15)° + (2x - 5)° + (3x + 75)° + (3x - 25)°$

$= 360°$

$2x° + 2x° + 3x° + 3x° = 360° - 15° + 5° - 75°$

$+ 25°$

$10x° = 300°$

$x° = 30°$

$\therefore x = 30$

(ii) Smallest interior angle

$= (2x - 5)°$

$= (2 \times 30 - 5)°$

$= 55°$

(iii) Largest interior angle gives the smallest exterior angle

Largest interior angle

$= (3x + 75)°$

$= (3 \times 30 + 75)°$

$= 165°$

Smallest exterior angle = 180° – 165° = 15°

17. **(i)** Sum of interior angles of a hexagon
$= (n - 2) \times 180°$
$= (6 - 2) \times 180°$
$= 720°$
$(2x + 17)° + (3x - 25)° + (2x + 49)° + (x + 40)° +$
$(4x - 17)° + (3x - 4)° = 720°$
$2x° + 3x° + 2x° + x° + 4x° + 3x°$
$= 720° - 17° + 25° - 49° - 40° + 17° + 4°$
$15x° = 660°$
$x° = 44°$
$\therefore x = 44$

(ii) Smallest interior angle of the hexagon
$= (x + 40)°$
$= (44 + 40)°$
$= 84°$

(iii) The largest interior angle gives the smallest exterior angle.
The largest interior angle
$= (4x - 17)°$
$= (4 \times 44 - 17)°$
$= 159°$
The smallest exterior angle $= 180° - 159° = 21°$

18. **(i)** The sum of exterior angles of a pentagon $= 360°$.
$2x° + (2x + 5)° + (3x + 10)° + (3x - 15)° + (x + 30)°$
$= 360°$
$2x° + 2x° + 3x° + 3x° + x°$
$= 360° - 5° - 10° + 15° - 30°$
$11x° = 330°$
$x° = 30°$
$\therefore x = 30$

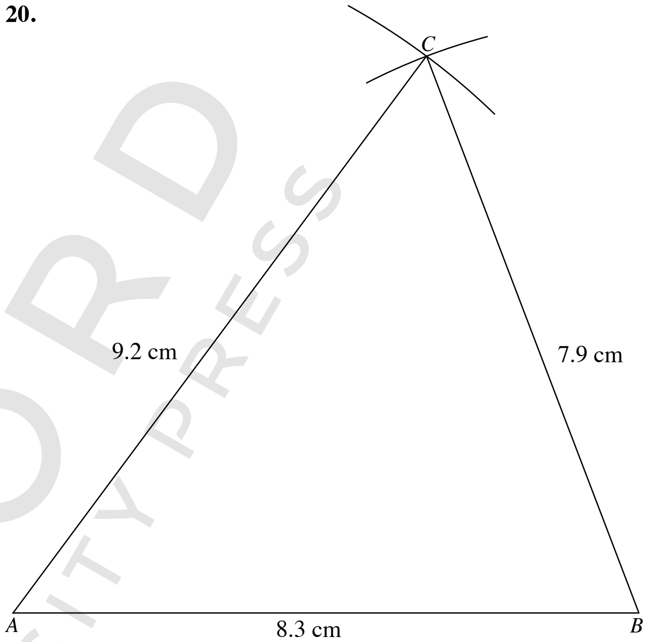
(ii) The largest exterior angle gives the smallest interior angle.
The largest exterior angle
$= (3x + 10)°$
$= (3 \times 30 + 10)°$
$= 100°$
The smallest interior angle
$= 180° - 100° = 80°$

(iii) The smallest exterior angle gives the largest interior angle.
Smallest exterior angle
$= 2x°$
$= 2(30°) = 60°$
The largest interior angle $= 180° - 60° = 120°$

19. **(i)** The sum of interior angles of a quadrilateral $= 360°$
30 parts $= 360°$
1 part $= 12°$
9 parts $= 12 \times 9 = 108°$
The largest interior angle $= 108°$.

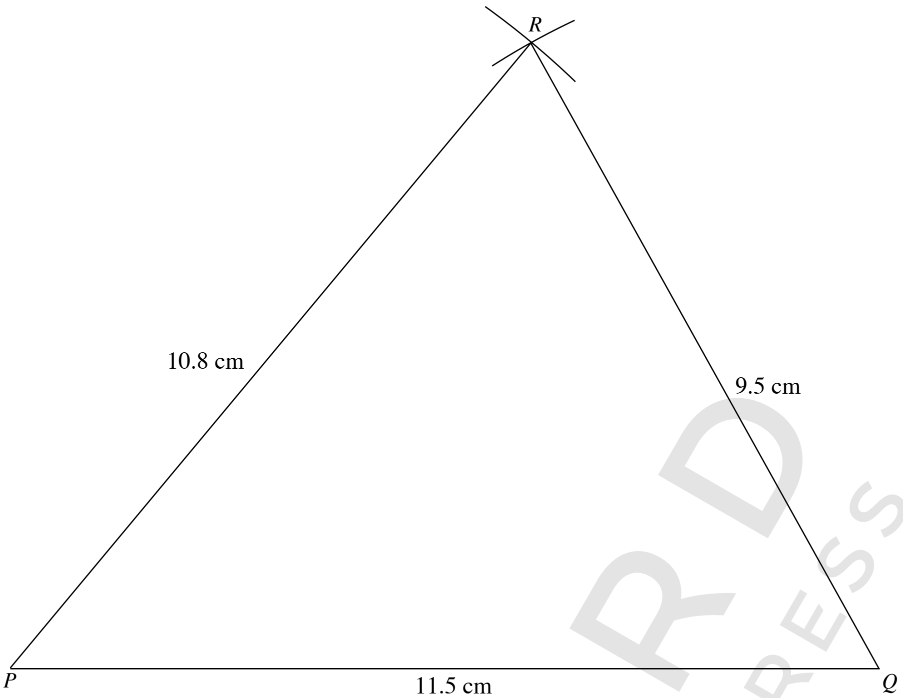
(ii) 6 parts $= 12° \times 6 = 72°$
The smallest interior angle $= 72°$.
The largest exterior angle
$= 180° - 72°$
$= 108°$

20.



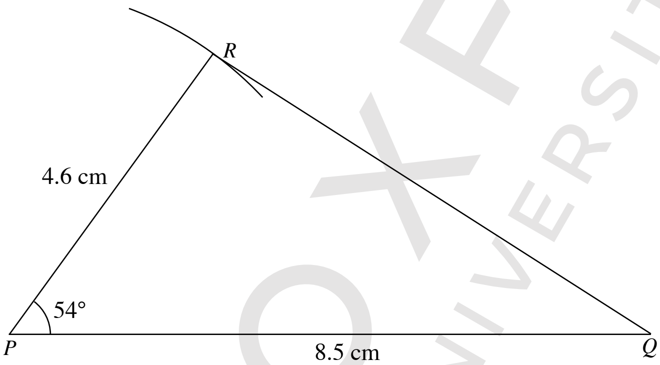
$\angle ABC = 69°$

21.



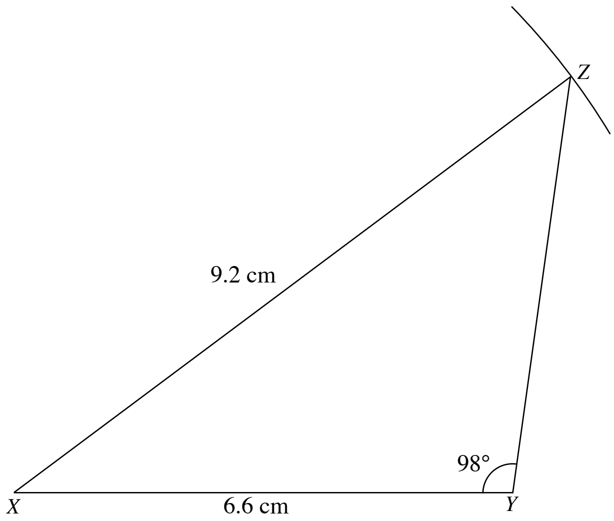
$\angle QPR = 50°$
$\angle PQR = 61°$

22.



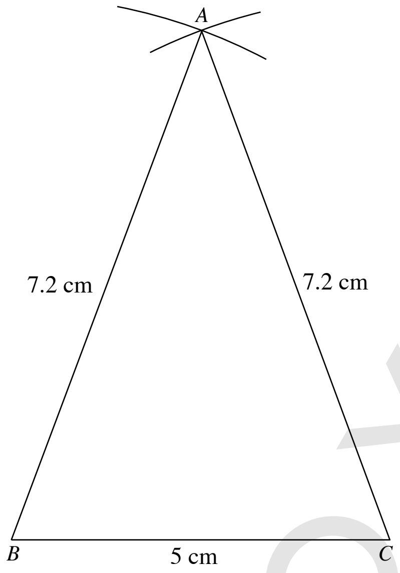
Length of QR = 6.9 cm.

23.



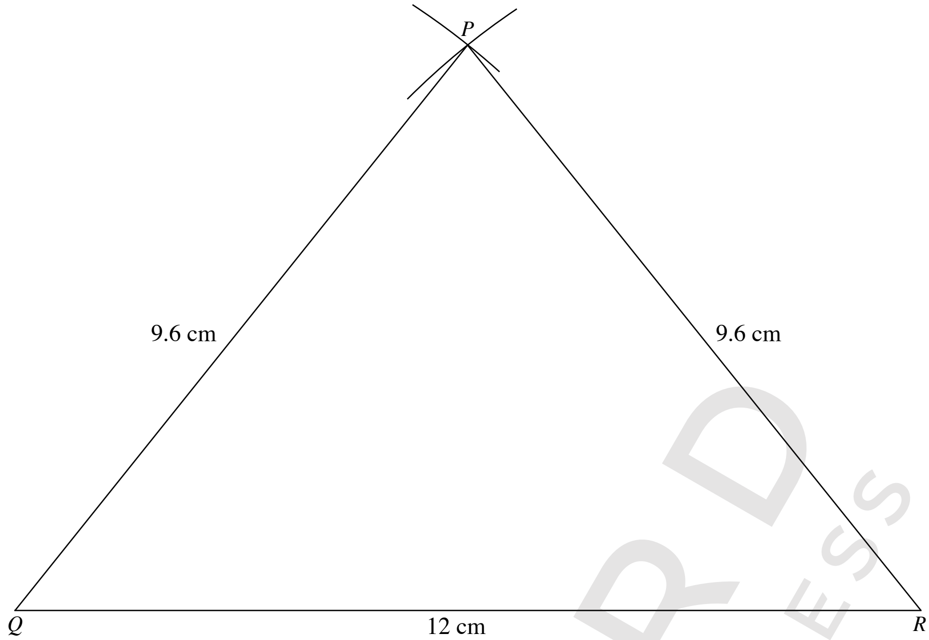
$\angle YXZ = 37°$

24.



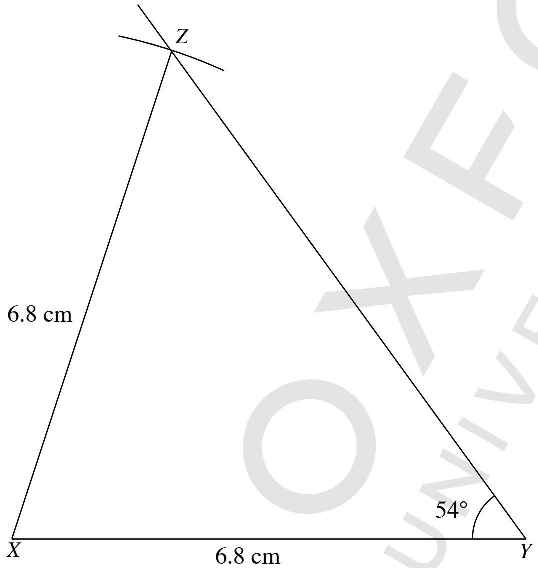
$\angle ABC = 70°$

25.



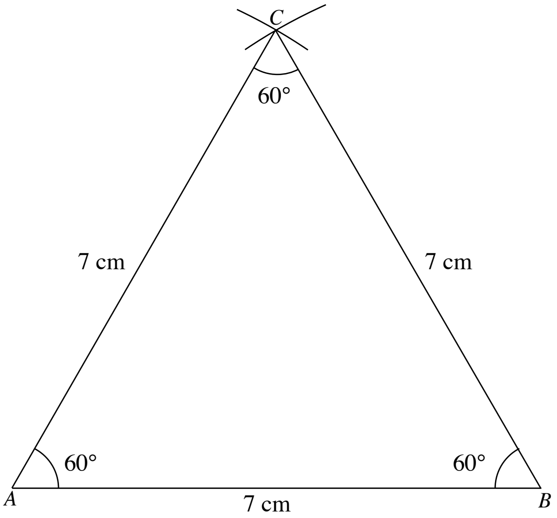
$\angle PQR = 51°$

26.

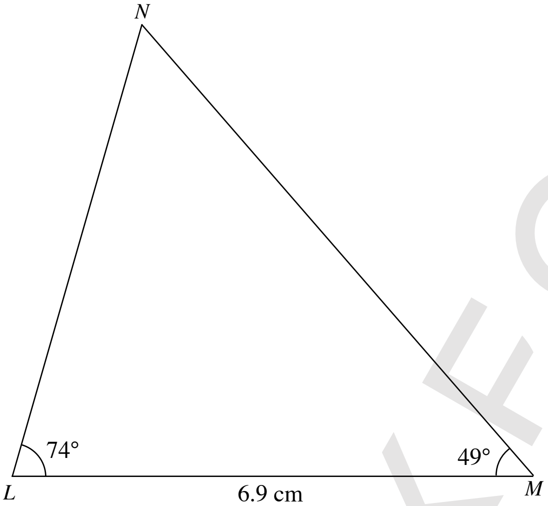


Length of YZ = 8.0 cm

27.



28.



Length of $LN = 6.2$ cm

Intermediate

29. (a) $y° = 61° + 59°$ (ext. ∠ of △)
$= 120°$
$\angle GDE = 61°$ (vert. opp. ∠s)
$x° = 69° + 61°$
$= 130°$ (ext. ∠ of △)
$\therefore x = 130$ and $y = 120$

(b) $x° = 110° + 40°$ (ext. ∠ of △)
$= 150°$
$\angle ADB + x° = 180°$ (adj. ∠s on a str. line)
$\angle ADB = 180° - x°$
$= 180° - 150°$
$= 30°$
$y° = 30° + 90°$ (ext. ∠ of △)
$= 120°$
$\therefore x = 150$ and $y = 120$

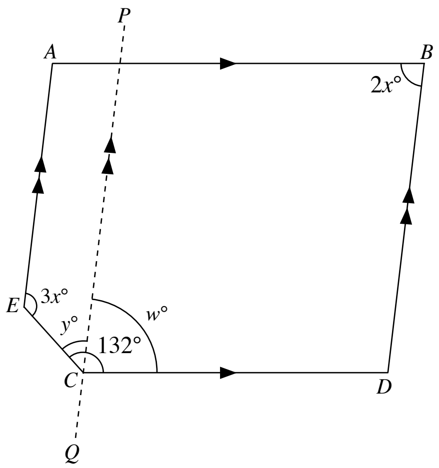
30. (a) $\angle BEF = 180° - 84° = 96°$ (adj. ∠s on a str. line)
Sum of angles in a quadrilateral is 360°.
$x° + 92° + 118° + 96° = 360°$
$x° = 360° - 92° - 118° - 96°$
$= 54°$
$y° + x° + 92° = 180°$ (∠ sum of △)
$y° = 180° - x° - 92°$
$= 180° - 54° - 92°$
$= 34°$
$\therefore x = 54$ and $y = 34$

(b) $\angle EBA = 53°$ (corr. ∠s, CD // AB)
$y° + 53° = 360°$ (∠s at a point)
$y° = 360° - 53°$
$= 307°$
$\angle FED = 53°$ (base ∠s of isos. △)
$x° = 53° + 53°$ (ext. ∠ of △, corr. ∠s)
$= 106°$
$\therefore x = 106$ and $y = 307$

(c) $x° + 25° + 121° = 180°$ (corr. ∠s, adj. ∠s on a str. line)
$x° = 180° - 25° - 121°$
$= 34°$
$y° + x° + 78° = 180°$ (∠ sum of △)
$y° = 180° - x° - 78°$
$= 180° - 34° - 78°$
$= 68°$
$\therefore x = 34$ and $y = 68$

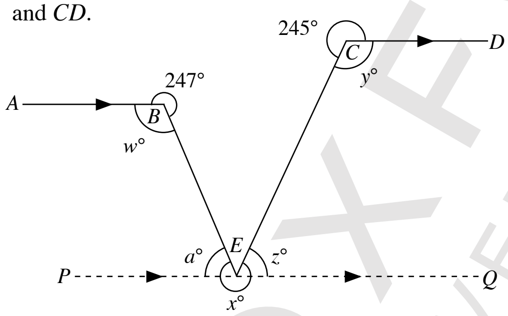
(d) $x° + 124° = 180°$ (corr. ∠s, adj. ∠s on a str. line)
$x° = 180° - 124°$
$= 56°$
$\angle ABD = 103°$ (corr. ∠s)
$y° + 103° = 180°$ (adj. ∠s on a str. line)
$y° = 180° - 103°$
$= 77°$
$z° + y° = 124°$ (ext. ∠ of △)
$z° = 124° - y°$
$= 124° - 77°$
$= 47°$
$\therefore x = 56$, $y = 77$ and $z = 47$

(e) Draw a line PQ through C that is parallel to AE and BD.



$w° = 2x°$ (opp. $\angle$s of //gram)
$y° = 132° - w°$
$= 132° - 2x°$
$3x° + y° = 180°$ (int. $\angle$s, EA // QP)
$3x° + 132 - 2x° = 180°$
$x° = 180° - 132°$
$= 48°$
$\therefore x = 48$

(f) Draw a line PQ through E that is parallel to AB and CD.



$w° + 247° = 360°$ ($\angle$s at a point)
$w° = 360° - 247°$
$= 113°$
$a° + w° = 180°$ (int. $\angle$s, AB // PQ)
$a° = 180° - w°$
$= 180° - 113°$
$= 67°$
$y° + 245° = 360°$ ($\angle$s at a point)
$y° = 360° - 245°$
$= 115°$
$z° + y° = 180°$ (int. $\angle$s, CD // PQ)
$z° = 180° - y°$
$= 180° - 115°$
$= 65°$
$x° = 180° + 67° + 65°$
$= 312°$
$\therefore x = 312$

31. Sum of angles in a triangle = 180°

$$(2x - 5)° + \left(3x - \frac{1}{2}\right)° + \left(30 - \frac{1}{2}x\right)° = 180°$$
$$2x° + 3x° - \frac{1}{2}x° = 180° + 5° + \frac{1}{2}° - 30°$$
$$4.5x° = 155.5°$$
$$x° = 34\frac{5}{9}°$$
$$\therefore x = 34\frac{5}{9}$$

32. (a) $64° + \angle ADC = 180°$ (int. $\angle$s, DC // AB)
$\angle ADC = 180° - 64°$
$= 116°$

$x° = \frac{1}{2} \times 116°$
$= 58°$
$y° = x°$ (alt. $\angle$s, DA // CB)
$= 58°$
$\therefore x = y = 58$

(b) $108° + \angle BAD = 180°$ (int. $\angle$s, BC // AD)
$\angle BAD = 180° - 108°$
$= 72°$

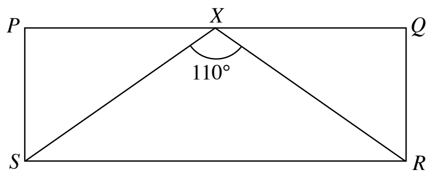
$x° = \frac{1}{2} \times 72°$
$= 36°$
$y° = x°$ (alt. $\angle$s, DC // AB)
$= 36°$
$\therefore x = y = 36$

(c) $(y - 5)° = 36°$ (alt. $\angle$s, DC // AB)
$y° = 36° + 5°$
$y° = 41°$
$x° = (y - 5)°$
$= (41 - 5)°$
$x° = 36°$
$\therefore x = 36$ and $y = 41$

(d) $(3x - 30)° = (2x + 15)°$ (opp. $\angle$s of //gram)
$3x° - 2x° = 15° + 30°$
$x° = 45°$
$(3x - 30)° + \angle BCD = 180°$ (int. $\angle$s, BA // CD)
$\angle BCD = 180° - (3x - 30)°$
$= 180° - (3 \times 45 - 30)$
$= 75°$

$y° = \frac{1}{2} \times 75°$
$= 37.5°$
$z° = y°$ (alt. $\angle$s, BA // CD)
$= 37.5°$
$\therefore x = 45$, $y = 37.5$ and $z = 37.5$

33.



(i) $\angle PXS = \dfrac{180° - 110°}{2}$

$= 35°$ (adj. $\angle$s on a str. line)

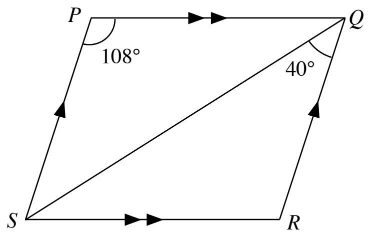
$\angle PSX = 180° - 90° - 35°$ ($\angle$sum of $\triangle$)

$= 55°$

(ii) $\angle XRS = \angle QXR$ (alt. $\angle$s PQ // SR)

$= 35°$

34.



(i) $\angle PSQ = 40°$ (alt. $\angle$s, PS // QR)

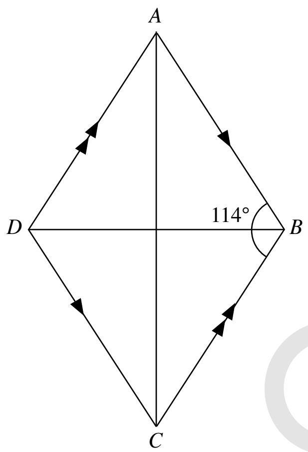
(ii) $\angle PSR = 180° - 108°$ (int. $\angle$s, PQ // SR)

$= 72°$

$\angle QSR = 72° - 40°$

$= 32°$

35.



(i) $\angle ABD = \dfrac{114°}{2}$

$= 57°$

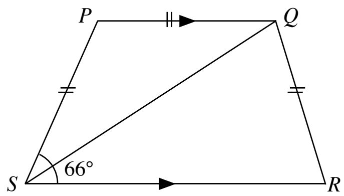
(ii) $\angle ACD = \angle ACB$

$= 180° - 90° - 57°$

(The diagonals of a rhombus bisect at 90°)

$= 33°$

36.



(i) $\angle QRS = 66°$ ($PS = QR$)

(ii) $\angle SPQ = 180° - 66°$ (int. $\angle$s, PQ // SR)

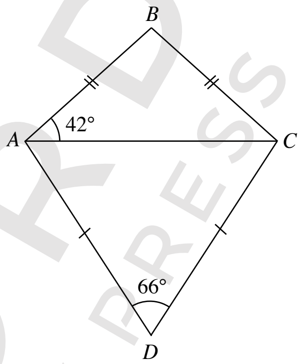
$= 114°$

$\angle PQS = \angle PSQ$

$= \dfrac{180° - 114°}{2}$ (base $\angle$s of isos. $\triangle PQS$)

$= 33°$

37.



(i) $\angle ACD = \dfrac{180° - 66°}{2}$ (base $\angle$s of isos. $\triangle ACD$)

$= 57°$

(ii) $\angle ABC = 180° - 42° - 42°$ (base $\angle$s of isos. $\triangle ABC$)

$= 96°$

38. (i) $115° + \angle PQR = 180°$ (int. $\angle$s, PQ // SR)

$\angle PQR = 180° - 115°$

$= 65°$

$\angle UQV = 65° - 35°$

$= 30°$

$\angle TVQ + 30° = 110°$ (ext. $\angle$ of $\triangle QUV$)

$\angle TVQ = 110° - 30°$

$= 80°$

(ii) $\angle SXY = 110°$ (corr. $\angle$s, TV // WY)

$110° + \angle SXW = 180°$ (adj. $\angle$s on a str. line)

$\angle SXW = 180° - 110°$

$= 70°$

39. (i) $66° + \angle ADC = 180°$ (int. $\angle$s, AB // DC)

$\angle ADC = 180° - 66°$

$= 114°$

$\angle CDQ + 145° + 114° = 360°$ ($\angle$s at a point)

$\angle CDQ = 360° - 145° - 114°$

$= 101°$

(ii) $114° + \angle BCD = 180°$ (int. $\angle$s, AD // BC)
$\angle BCD = 180° - 114°$
$= 66°$

(iii) $\angle DCP = \angle CDQ = 101°$ (alt. $\angle$s, PC // DQ)
$\angle PCB = 101° - 66°$
$= 35°$

40. (i) $\angle DAC = 33°$
$\angle QBC = 66°$ (corr. $\angle$s, AD // BC)

(ii) $66° + \angle ADC = 180°$ (int. $\angle$s, AB // DC)
$\angle ADC = 180° - 66°$
$= 114°$
$\angle ADB = 114° \div 2 = 57°$
$\angle DBC = \angle ADB$ (alt. $\angle$s, AD // BC)
$= 57°$

(iii) $\angle BCD = 66°$ (opp. $\angle$s in a //gram)
$72° + 66° +$ Reflex $\angle BCR = 360°$ ($\angle$s at a point)
Reflex $\angle BCR = 360° - 72° - 66°$
$= 222°$

41. (i) $\angle AEB = 180° - 52° - 90°$ ($\angle$ sum of $\triangle$)
$= 38°$
$\angle PEQ = \angle AEB$ (vert. opp. $\angle$s)
$= 38°$

(ii) $\angle QED = 90° - 38° = 52°$
$\angle EDR = 180° - 52°$ (int. $\angle$s, QE // RD)
$= 128°$

(iii) $\angle EDC = 360° - 126° - 128°$ ($\angle$s at a point)
$= 106°$
$\angle BCD + 106° = 180°$ (int. $\angle$s, ED // AC)
$\angle BCD = 180° - 106°$
$= 72°$

42. (i) $\angle ABQ = 45° - 21°$
$= 24°$
$\angle BAQ = \dfrac{180° - 24°}{2}$ (base $\angle$s of isos. $\triangle ABQ$)
$= 78°$

(ii) Since $BQ = BA$, $BQ = BC$,
$\angle ABC = 45° + 21°$
$= 66°$
$\angle BCQ = \dfrac{180° - 66°}{2}$ (base $\angle$s of isos. $\triangle QBC$)
$= 57°$
$\angle DCQ = 90° - 57°$
$= 33°$

(iii) $\angle DPC = 180° - 45° - 33°$ ($\angle$ sum of $\triangle$)
$= 102°$
$\angle QPB = 102°$ (vert. opp. $\angle$s)

43. (i) $\angle QAD = 60°$ ($\angle$s of equilateral $\triangle$)
$\angle BAD + 90° + 135° + 60° = 360°$ ($\angle$s at a point)
$\angle BAD = 360° - 90° - 135° - 60°$
$= 75°$

(ii) Sum of interior angles of a quadrilateral = 360°.
$\angle CDA + 106° + 100° + 75° = 360°$ ($\angle$s at a point)
$\angle CDA = 360° - 106° - 100° - 75°$
$= 79°$

(iii) $\angle QDP + 60° + 79° = 180°$ (adj. $\angle$s on a str. line)
$\angle QDP = 180° - 60° - 79°$
$= 41°$

44. (i) $\angle ABC = 180° - 68° - 68°$ (base $\angle$s of isos. $\triangle ABC$)
$= 44°$

(ii) $\angle ACD = 68°$
$\angle ADC = 180° - 68° - 68°$ ($\angle$ sum of $\triangle$)
$= 44°$
$60° + 90° + \angle ADP = 180°$ ($\angle$ sum of $\triangle$)
$\angle ADP = 180° - 90° - 60°$
$= 30°$
$\angle PDQ + \angle ADP = \angle ADC$
$\angle PDQ = \angle ADC - \angle ADP$
$= 44° - 30°$
$= 14°$

(iii) $\angle DQR = \angle PDQ + \angle DPR$
$= 14° + 90°$
$= 104°$

45. (i) $\angle TBD = 180° - 81°$ (int. $\angle$s, BD // TE)
$= 99°$
$\angle DBC = 61°$ (alt. $\angle$s, ED // BC)
$\angle ABT = 180° - 99° - 61°$ (adj. $\angle$s on a str. line)
$= 20°$

(ii) $\angle TED = 180° - 61°$ (int. $\angle$s, DB // ET)
$= 119°$

(iii) $\angle BCD = 180° - 61° - 61°$
$= 58°$

46. (i) $\angle BAD = 180° - 88°$ (int. $\angle$s, BC // AD)
$= 92°$
$\angle DAE = 180° - 92°$ (adj. $\angle$s on a str. line)
$= 88°$
$\angle AED = \dfrac{180° - 88°}{2}$ (base $\angle$s of isos. $\triangle ADE$)
$= 46°$

(ii) $\angle FAB = 180° - 162°$ (adj. $\angle$s on a str. line)
$= 18°$
$\angle FAD = \angle FAB + \angle BAD$
$= 18° + 92°$
$= 110°$

(iii) $\angle ADC = 88°$ (opp. $\angle$s in a //gram)

Sum of angles in a quadrilateral = 360°

$\angle FCD + 48° + 110° + 88° = 360°$

$\angle FCD = 360° - 48° - 110° - 88°$

$= 114°$

$\angle BCF = \angle FCD - \angle BCD$

$= 114° - 92°$

$= 22°$

47. Let the number of sides of the polygon be n.

Sum of interior angles = $(n - 2) \times 180°$

Sum of exterior angles = 360°

$(n - 2) \times 180° = 2 \times 360°$

$180n - 360 = 720$

$180n = 720 + 360$

$= 1080$

$n = 6$

$\therefore$ The number of sides of the polygon is 6.

48. Let the number of sides of the regular polygon be n.

Size of each interior angle = $\dfrac{(n-2) \times 180°}{n}$

Size of each exterior angle = $\dfrac{360°}{n}$

$\dfrac{(n-2) \times 180°}{n} = 35 \times \dfrac{360°}{n}$

$(n - 2) \times 180° = 35 \times 360°$

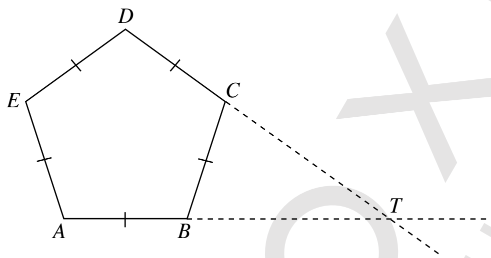
$180n - 360 = 12\ 600$

$180n = 12\ 600 + 360$

$= 12\ 960$

$n = 72$

49.



Size of each exterior angle of the pentagon = $\dfrac{360°}{5}$

$= 72°$

$\angle CBT = \angle BCT = 72°$

$\angle BTC = 180° - 72° - 72°$ ($\angle$ sum of $\triangle$)

$= 36°$

50. (i) Size of each interior angle = $\dfrac{(12-2) \times 180°}{12}$

$= 150°$

$\therefore \angle ABC = 150°$

(ii) $\angle BCA = \dfrac{180° - 150°}{2} = 15°$

$\angle ACD + \angle BCA = \angle BCD = 150°$

$\angle ACD = 150° - \angle BCA$

$= 150° - 15°$

$= 135°$

51. (a) Let the number of sides of the polygon be n.

Sum of interior angles = $(n - 2) \times 180°$

$(n - 2) \times 180° = 124° + (n - 1) \times 142°$

$180n - 360 = 124 + 142n - 142$

$180n - 142n = 124 - 142 + 360$

$38n = 342$

$n = 9$

$\therefore$ The number of sides of the polygon is 9.

(b) Size of each angle in a pentagon $ABCDE$

$= \dfrac{(5-2) \times 180°}{5}$

$= 108°$

Size of each angle in a hexagon $CDZYXW$

$= \dfrac{(6-2) \times 180°}{6}$

$= 120°$

(i) $\angle WCD$ = size of each angle in a hexagon

$= 120°$

(ii) $\angle BCD$ = size of each angle in a pentagon

$= 108°$

(iii) Since $CB = CW$, $\triangle BCW$ is an isosceles triangle.

$\angle BCW = 360° - 108° - 120°$ ($\angle$s at a point)

$= 132°$

$\angle CBW = \dfrac{180° - 132°}{2}$ (base $\angle$s of isos. $\triangle BCW$)

$= 24°$

52. (i) $\angle CBA = 180° - 18°$ (adj. $\angle$s on a str. line)

$= 162°$

(ii) Size of each interior angle = $\dfrac{(n-2) \times 180°}{n}$

$162 = \dfrac{(n-2) \times 180°}{n}$

$162n = (n - 2) \times 180°$

$162n = 180n - 360$

$180n - 162n = 360$

$18n = 360$

$n = 20$

$\therefore$ The value of n is 20.

(iii) $\angle BCY = \dfrac{180° - 162°}{2}$

$= 9°$

$\angle CBY = (360° - 162° - 162°) \div 2$

$= 18°$

$\angle BYC = 180° - 18° - 9°$ ($\angle$ sum of $\triangle$)

$= 153°$

53. (i) Size of each interior angle $= \dfrac{(n-2) \times 180°}{n}$.

$174° = \dfrac{(n-2) \times 180°}{n}$

$174n = (n-2) \times 180$

$174n = 180n - 360$

$180n - 174n = 360$

$6n = 360$

$n = 60$

$\therefore$ The value of n is 60.

(ii) $\angle PBC = \dfrac{(5-2) \times 180°}{5}$ (base $\angle$s of isos. $\triangle PBQ$)

$= 108°$

$\angle ABP = 360° - 108° - 174°$ ($\angle$s at a point)

$= 78°$

(iii) $\angle PBQ = \dfrac{180° - 108°}{2}$

$= 36°$

$\angle QBC = 108° - 36°$

$= 72°$

$\angle BQC = 180° - 72° - 72°$ (base $\angle$s of isos. $\triangle BQC$)

$= 36°$

(iv) $\angle DCR = 360° - 108° - 174°$ ($\angle$s at a point)

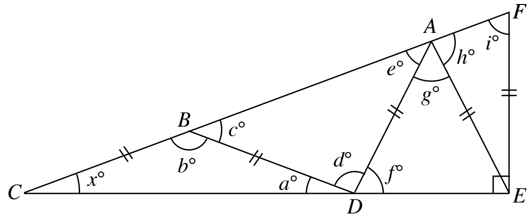
$= 78°$

$\angle CDR = \dfrac{180° - 78°}{2}$ (base $\angle$s of isos. $\triangle CDR$)

$= 51°$

Advanced

54.



(i) For $\triangle BCD$,

$a° = x°$ (base $\angle$s of isos. $\triangle BCD$)

$b° = 180° - x° - x°$ ($\angle$ sum of $\triangle BCD$)

$= 180° - 2x°$

For $\triangle ADB$,

$c° = 180° - b°$ (adj. $\angle$s on a str. line)

$= 180° - (180° - 2x°)$

$= 180° - 180° + 2x°$

$= 2x°$

$e° = c°$ (base $\angle$s of isos. $\triangle ADB$)

$d° = 180° - c° - e°$ ($\angle$ sum of $\triangle ABD$)

$= 180° - 2x° - 2x°$

$= 180° - 4x°$

For $\triangle ADE$,

$f° = 180° - x° - (180° - 4x°)$ (adj. $\angle$s on a str. line)

$= 180° - x° - 180° + 4x°$

$= 3x°$

$g° = 180° - 3x° - 3x°$ ($\angle$ sum of $\triangle ADE$)

$= 180° - 6x°$

For $\triangle AEF$,

$h° = 180° - 2x° - (180° - 6x°)$ (adj. $\angle$s on a str. line)

$= 180° - 2x° - 180° + 6x°$

$= 4x°$

$i° = h° = 4x°$ (base $\angle$s of isos. $\triangle AEF$)

$i° + x° + 90° = 180°$ ($\angle$ sum of $\triangle CEF$)

$4x° + x° + 90° = 180°$

$5x° = 180° - 90° = 90°$

$x° = 18°$

$\therefore x = 18$

(ii) Let n be the number of isosceles triangles that can be formed.

From **(i)**,

$(n+1)x° + 90° = 180°$

When $x° = 5°$,

$(n+1)5° + 90° = 180°$

$5(n+1) = 90$

$n + 1 = 18$

$n = 17$

$\therefore$ There are 17 isosceles triangles that can be formed when $x = 5$.

55. (i) Let polygon A have a sides and polygon B have b sides.

$$\frac{360}{a} + \frac{360}{b} = 80$$
$$360(a + b) = 80ab$$
$$9(a + b) = 2ab$$
$$9a + 9b = 2ab$$
$$9a = 2ab - 9b$$
$$b = \frac{9a}{2a - 9}$$

$\therefore$ A possible solution is polygon A has 5 sides and polygon B has 45 sides.

(ii) Sum of exterior angles of any polygon = 360°.
When the exterior angle of their shared side decreases, the corresponding exterior angle of each polygon decreases.

$$\text{Number of sides} = \frac{360°}{\text{size of each exterior angle}}$$

$\therefore$ Number of sides increases as size of each interior angle in both polygons decreases.

New Trend

56. Let the first angle be $x°$.

$$x + (x - 10) + 4(x - 10) + \left(x + \frac{120}{100}x\right) = 360$$
$$x + x - 10 + 4x - 40 + 2.2x = 360$$
$$8.2x = 410$$
$$x = 50$$

The angles of the quadrilateral are 50°, 40°, 160° and 110°.

57. Sum of interior angles of a polygon with 6 sides
$= (n - 2) \times 180°$
$= (6 - 2) \times 180°$
$= 720°$

$$d° + 125° + d° + 3d° + 70° + 110° = 720°$$
$$d° + d° + 3d° = 720° - 125° - 70° - 110°$$
$$5d° = 415°$$
$$d° = 83°$$

$\therefore d = 83$

58. Size of each interior angle of the decagon
$= \frac{(10 - 2) \times 180°}{10}$
$= 144°$

Size of each interior angle of the hexagon
$= \frac{(6 - 2) \times 180°}{6}$
$= 120°$

$x° = 360° - 144° - 120°$ ($\angle$s at a point)
$= 96°$

$\therefore x = 96$

59. $x° + 63° = 180°$ (int. $\angle$s, $AD \mathbin{//} BC$)
$x° = 180° - 63°$
$= 117°$
$y° = 61°$ (alt. $\angle$s, $DC \mathbin{//} AB$)
$AD = BC = 7.5$ cm
$z = 7.5$
$\therefore x = 117$, $y = 61$ and $z = 7.5$

60. (a) Size of each interior angle of a regular 24-sided polygon $= \frac{(24 - 2) \times 180°}{24}$
$= 165°$

(b) Let the number of sides of the polygon be n.
Sum of interior angles $= (n - 2) \times 180°$

$$(n - 2) \times 180° = 172° + 2(158°) + (n - 3)p°$$
$$488 + (n - 3)p = 180n - 360$$
$$(n - 3)p = 180n - 848$$
$$p = \frac{180n - 848}{n - 3}$$

61. (a) Size of each interior angle $= \frac{(n - 2) \times 180°}{n}$

$$150° = \frac{(n - 2) \times 180°}{n}$$
$$150n = (n - 2) \times 180$$
$$150n = 180n - 360$$
$$180n - 150n = 360$$
$$30n = 360$$
$$n = 12$$

(b) Size of each exterior angle $= \frac{360°}{n}$
$= \frac{360°}{9}$
$= 40°$

62. Let the number of sides of the regular polygon be n.

Size of each interior angle $= \frac{(n - 2) \times 180°}{n}$

$$\frac{(n - 2) \times 180°}{n} = 165.6°$$
$$(n - 2) \times 180 = 165.6n$$
$$180n - 165.6n = 2 \times 180$$
$$14.4n = 360$$
$$n = 25$$

Chapter 11 Symmetry

Basic

1. **(a)** False

(b) False

(c) True

(d) True

(e) False

(f) True

(g) True

(h) False

(i) True

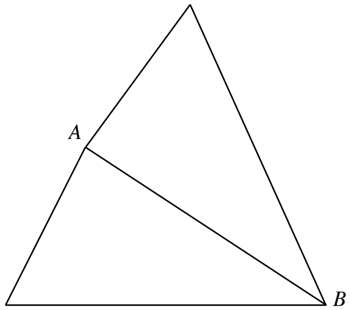
(j) False

(k) False

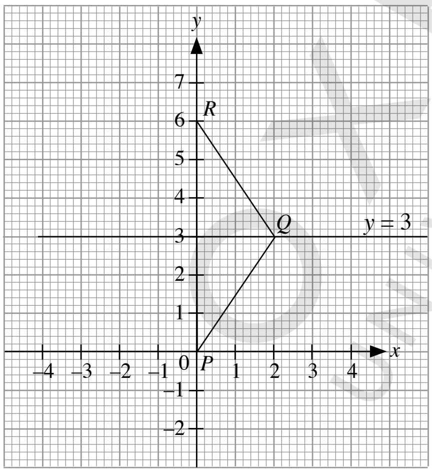
(l) False

2. **(a)** An equilateral triangle has 3 lines of symmetry.

(b)



3. **(i)**



(ii) The equation of the line of symmetry is $y = 3$.

Chapter 12 Perimeter, Area, Surface Area, and Volume

Basic

1. Circumference of a circle = $2\pi r$

Area of a circle = πr^2

	Diameter	Radius	Circumference	Area
(a)	2×10 $= 20$ cm	10 cm	$2 \times 3.142 \times 10$ $= 62.8$ cm (to 3 s.f.)	3.142×10^2 $= 314$ cm^2 (to 3 s.f.)
(b)	2×0.7495 $= 0.150$ m (to 3 s.f.)	$0.471 \div (2 \times 3.142)$ $= 0.07495$ $= 0.0750$ m (to 3 s.f.)	0.471 m	3.142×0.07495^2 $= 0.0177$ m^2 (to 3 s.f.)
(c)	1.2 m	$1.2 \div 2$ $= 0.6$ m	$2 \times 3.142 \times 0.6$ $= 3.77$ m (to 3 s.f.)	3.142×0.6^2 $= 1.13$ m^2 (to 3 s.f.)
(d)	3.999×2 $= 8.00$ cm (to 3 s.f.)	$\sqrt{50.24 \div 3.142}$ $= 3.999$ cm $= 4.00$ cm (to 3 s.f.)	$2 \times 3.142 \times 3.999$ $= 25.1$ cm (to 3 s.f.)	50.24 cm^2
(e)	2×11.996 $= 24.0$ cm (to 3 s.f.)	$\sqrt{452.16 \div 3.142}$ $= 11.996$ cm $= 12.0$ cm (to 3 s.f.)	$2 \times 3.142 \times 11.996$ $= 75.4$ cm (to 3 s.f.)	452.16 cm^2
(f)	2×14 $= 28$ cm	14 cm	$2 \times 3.142 \times 14$ $= 88.0$ cm	3.142×14^2 $= 616$ cm^2 (to 3 s.f.)
(g)	2×4.2 $= 8.4$ cm	4.2 cm	$2 \times 3.142 \times 4.2$ $= 26.4$ cm (to 3 s.f.)	3.142×4.2^2 $= 55.4$ cm^2 (to 3 s.f.)
(h)	2×19.987 $= 40.0$ m (to 3 s.f.)	$125.6 \div (2 \times 3.142)$ $= 19.987$ m $= 20.0$ m (to 3 s.f.)	125.6 m	3.142×19.987^2 $= 1260$ m^2 (to 3 s.f.)
(i)	84 mm	$84 \div 2$ $= 42$ mm	$2 \times 3.142 \times 42$ $= 264$ mm (to 3 s.f.)	3.142×42^2 $= 5540$ mm^2 (to 3 s.f.)
(j)	2×21.0057 $= 42.0$ cm (to 3 s.f.)	$132 \div (2 \times 3.142)$ $= 21.0057$ cm $= 21.0$ cm (to 3 s.f.)	132 cm	3.142×21.0057^2 $= 1390$ cm^2 (to 3 s.f.)
(k)	2×12.4920 $= 25.0$ cm (to 3 s.f.)	$78.5 \div (2 \times 3.142)$ $= 12.4920$ cm $= 12.5$ cm (to 3 s.f.)	78.5 cm	3.142×12.4920^2 $= 490$ cm^2 (to 3 s.f.)
(l)	56 cm	$56 \div 2$ $= 28$ cm	$2 \times 3.142 \times 28$ $= 176$ cm (to 3 s.f.)	3.142×28^2 $= 2460$ cm^2 (to 3 s.f.)
(m)	2×38.9752 $= 78.0$ mm (to 3 s.f.)	$244.92 \div (2 \times 3.142)$ $= 38.9752$ mm $= 39.0$ mm (to 3 s.f.)	244.92 mm	3.142×38.9752^2 $= 4770$ mm^2 (to 3 s.f.)
(n)	60 cm	$60 \div 2$ $= 30$ cm	$2 \times 3.142 \times 30$ $= 189$ cm (to 3 s.f.)	3.142×30^2 $= 2830$ cm^2 (to 3 s.f.)
(o)	2×4.9984 $= 10.0$ cm (to 3 s.f.)	$\sqrt{78.5 \div 3.142}$ $= 4.9984$ cm $= 5.00$ cm	$2 \times 3.142 \times 4.9984$ $= 31.4$ cm (to 3 s.f.)	78.5 cm^2

2. **(a)** **(i)** Perimeter of figure

$$= \left[\frac{1}{2} \times 2 \times 3.142 \times \left(\frac{49}{2}\right)\right] + 49$$
$$= 76.979 + 49$$
$$= 125.979$$
$$= 126 \text{ cm (to 3 s.f.)}$$

(ii) Area of figure

$$= \frac{1}{2} \times 3.142 \times \left(\frac{49}{2}\right)^2$$
$$= 942.992\ 75$$
$$= 943 \text{ cm}^2 \text{ (to 3 s.f.)}$$

(b) **(i)** Perimeter of figure

$$= \left[\frac{1}{2} \times 2 \times 3.142 \times \left(\frac{21}{2}\right)\right] + 20 + 21 + 20$$
$$= 32.991 + 61$$
$$= 93.991$$
$$= 94.0 \text{ cm (to 3 s.f.)}$$

(ii) Area of figure

$$= (20 \times 21) + \left[\frac{1}{2} \times 3.142 \times \left(\frac{21}{2}\right)^2\right]$$
$$= 420 + 173.202\ 75$$
$$= 593.202\ 75$$
$$= 593 \text{ cm}^2 \text{ (to 3 s.f.)}$$

(c) **(i)** Perimeter of figure

$$= \left[\frac{1}{2} \times 2 \times 3.142 \times \left(\frac{21}{2}\right)\right] + \left[\frac{1}{2} \times 2 \times 3.142 \times \left(\frac{14}{2}\right)\right] + 21 + 14$$
$$= 32.991 + 21.994 + 21 + 14$$
$$= 89.985$$
$$= 90.0 \text{ cm (to 3 s.f.)}$$

(ii) Area of figure

$$= (14 \times 21) + \left[\frac{1}{2} \times 3.142 \times \left(\frac{21}{2}\right)^2\right] + \left[\frac{1}{2} \times 3.142 \times \left(\frac{14}{2}\right)^2\right]$$
$$= 294 + 173.202\ 75 + 76.979$$
$$= 544.181\ 75$$
$$= 544 \text{ cm}^2 \text{ (to 3 s.f.)}$$

(d) **(i)** Perimeter of figure

$$= \left[2 \times 3.142 \times \left(\frac{28}{2}\right)\right] + 16 + 16$$
$$= 87.976 + 16 + 16$$
$$= 119.976$$
$$= 120 \text{ cm (to 3 s.f.)}$$

(ii) Area of figure

$= 28 \times 16$

$= 448 \text{ cm}^2$

(Note: The semicircle removed from the rectangle can be replaced by the semicircle that is placed beside the rectangle. Therefore, the area of the figure is that of a rectangle of 28 cm by 16 cm.)

3. **(i)** Perimeter of the shaded region

$= 40 + 40 + \left[2 \times 3.142 \times \frac{28}{2}\right]$

$= 80 + 87.976$

$= 167.976$

$= 168$ cm (to 3 s.f.)

(ii) Area of the shaded region

$= (40 \times 28) - \left[3.142 \times \left(\frac{28}{2}\right)^2\right]$

$= 1120 - 615.832$

$= 504.168$

$= 504 \text{ cm}^2$ (to 3 s.f.)

4. **(i)** Perimeter of quadrant

$= \left(\frac{1}{4} \times 2 \times 3.142 \times 10\right) + 10 + 10$

$= 15.71 + 20$

$= 35.71$

$= 35.7$ cm (to 3 s.f.)

(ii) Area of quadrant

$= \frac{1}{4} \times 3.142 \times 10^2$

$= 78.55$

$= 78.6 \text{ cm}^2$ (to 3 s.f.)

5. **(a)** Area of circle with radius 10 cm

$= 3.142 \times 10^2$

$= 314.2 \text{ cm}^2$

Area of circle with radius 6 cm

$= 3.142 \times 6^2$

$= 113.112 \text{ cm}^2$

Area of shaded region

$= 314.2 - 113.112$

$= 201.088$

$= 201 \text{ cm}^2$ (to 3 s.f.)

(b) Area of circle of radius 10 cm

$= 3.142 \times 10^2$

$= 314.2 \text{ cm}^2$

Area of square

$= 14.14 \times 14.14$

$= 199.9396 \text{ cm}^2$

Area of shaded region

$= 314.2 - 199.9396$

$= 114.2604$

$= 114 \text{ cm}^2$ (to 3 s.f.)

(c) Area of circle with diameter 32 cm

$= 3.142 \times \left(\frac{32}{2}\right)^2$

$= 804.352 \text{ cm}^2$

Area of circle with diameter 20 cm

$= 3.142 \times \left(\frac{20}{2}\right)^2$

$= 314.2 \text{ cm}^2$

Area of shaded region

$= 804.352 - 314.2$

$= 490.152$

$= 490 \text{ cm}^2$ (to 3 s.f.)

(d) Area of square $= 16 \times 16 = 256 \text{ cm}^2$

Area of circle of diameter 16 cm

$= 3.142 \times \left(\frac{16}{2}\right)^2$

$= 201.088 \text{ cm}^2$

Area of shaded region

$= 256 - 201.088$

$= 54.912$

$= 54.9 \text{ cm}^2$ (to 3 s.f.)

Intermediate

6. **(a)** The base is a triangle with height 12 cm and base length 16 cm.

Base area = area of triangle

$= \frac{1}{2} \times 12 \times 16$

$= 96 \text{ cm}^2$

Volume of prism = base area × height

$= 96 \times 14$

$= 1344 \text{ cm}^3$

Total surface area of prism

$= 96 + 96 + (16 \times 14) + (12 \times 14) + (14 \times 20)$

$= 864 \text{ cm}^2$

(b) The shape of the base is a cross.

Base area

$= (14 \times 14) - 4(5 \times 5)$

$= 96 \text{ cm}^2$

Volume of prism = base area × height

$= 96 \times 3$

$= 288 \text{ cm}^3$

Total surface area of solid

$= (2 \times 96) + 8(5 \times 3) + 4(3 \times 4)$

$= 192 + 120 + 48$

$= 360 \text{ cm}^2$

(c) The base is a triangle with height 8 cm and base length 6 cm.

Base area = $\frac{1}{2} \times 8 \times 6$

$= 24 \text{ cm}^2$

Volume of prism = base area × height

$= 24 \times 14$

$= 336 \text{ cm}^3$

Total surface area of the solid

$= 24 + 24 + (10 \times 14) + (14 \times 8) + (6 \times 14)$

$= 48 + 140 + 112 + 84$

$= 384 \text{ cm}^2$

(d) The base is an inverted L-shape.

Base area = $(21 \times 15) - (9 \times 7)$

$= 315 - 63$

$= 252 \text{ cm}^2$

Volume of prism = base area × height

$= 252 \times 10$

$= 2520 \text{ cm}^3$

Total surface area of the solid

$= (252 \times 2) + 2(6 \times 10) + 2(7 \times 10) + (9 \times 10) + (21 \times 10) + 2(15 \times 10)$

$= 504 + 120 + 140 + 90 + 210 + 300$

$= 1364 \text{ cm}^2$

7. Volume of a closed cylinder = $\pi r^2 h$

Total surface area of closed cylinder = $2\pi r^2 + 2\pi rh$

	Diameter	**Radius**	**Height**	**Volume**	**Total Surface Area**
(a)	24×2 $= 48$ cm	24 cm	21 cm	$3.142 \times (24)^2 \times 21$ $= 38\ 000 \text{ cm}^3$ (to 3 s.f.)	$(2 \times 3.142 \times (24)^2)$ $+ (2 \times 3.142 \times 24 \times 21)$ $= 3619.584 + 3167.136$ $= 6790 \text{ cm}^2$ (to 3 s.f.)
(b)	1.45×2 $= 2.9$ cm	1.45 cm	1.4 cm	$3.142 \times (1.45)^2 \times 1.4$ $= 9.25 \text{ cm}^3$ (to 3 s.f.)	$(2 \times 3.142 \times (1.45)^2)$ $+ (2 \times 3.142 \times 1.45 \times 1.4)$ $= 13.212\ 11 + 12.756\ 52$ $= 26.0 \text{ cm}^2$ (to 3 s.f.)
(c)	28×2 $= 56$ cm	0.28 m = 28 cm	45 cm	$3.142 \times (28)^2 \times 45$ $= 111\ 000 \text{ cm}^3$ (to 3 s.f.)	$(2 \times 3.142 \times (28)^2)$ $+ (2 \times 3.142 \times 28 \times 45)$ $= 4926.656 + 7917.84$ $= 12\ 800 \text{ cm}^2$ (to 3 s.f.)
(d)	18.2×2 $= 36.4$ cm	182 mm = 18.2 cm	7.5 cm	$3.142 \times (18.2)^2 \times 7.5$ $= 7810 \text{ cm}^3$ (to 3 s.f.)	$(2 \times 3.142 \times (18.2)^2)$ $+ (2 \times 3.142 \times 18.2 \times 7.5)$ $= 2081.512\ 16 + 857.766$ $= 2940 \text{ cm}^2$ (to 3 s.f.)
(e)	4.998×2 $= 10.0$ cm (to 3 s.f.)	$\sqrt{(2826) \div (3.142 \times 36)}$ $= 4.998$ $= 5.00$ cm (to 3 s.f.)	36 cm	2826 cm^3	$(2 \times 3.142 \times (4.998)^2)$ $+ (2 \times 3.142 \times 4.998 \times 36)$ $= 156.97 + 1130.67$ $= 1290 \text{ cm}^2$ (to 3 s.f.)
(f)	1.118×2 $= 2.236$ cm	$\sqrt{(30.615) \div (3.142 \times 7.8)}$ $= 1.118$ cm $= 1.12$ cm (to 3 s.f.)	7.8 cm	30.615 cm^3	$(2 \times 3.142 \times (1.118)^2)$ $+ (2 \times 3.142 \times 1.118 \times 7.8)$ $= 7.854\ 52 + 54.799$ $= 62.7 \text{ cm}^2$ (to 3 s.f.)
(g)	19.994×2 $= 40.0$ cm (to 3 s.f.)	$\sqrt{(8164) \div (3.142 \times 6.5)}$ $= 19.994$ cm $= 20.0$ cm (to 3 s.f.)	65 mm = 6.5 cm	8164 cm^3	$(2 \times 3.142 \times (19.994)^2)$ $+ (2 \times 3.142 \times 19.994 \times 6.5)$ $= 2512.092 + 816.6749$ $= 3330 \text{ cm}^2$ (to 3 s.f.)
(h)	5.6×2 $= 11.2$ cm	5.6 cm	$532 \div (3.142 \times 5.6^2)$ $= 5.3992$ cm $= 5.40$ cm (to 3 s.f.)	532 cm^3	$(2 \times 3.142 \times (5.6)^2)$ $+ (2 \times 3.142 \times 5.6 \times 5.3992)$ $= 197.066\ 24 + 190.00$ $= 387 \text{ cm}^2$ (to 3 s.f.)
(i)	2.65×2 $= 5.3$ cm	2.65 cm	$20.74 \div (3.142 \times 2.65^2)$ $= 0.940$ cm (to 3 s.f.)	20.74 cm^3	$(2 \times 3.142 \times (2.65)^2)$ $+ (2 \times 3.142 \times 2.65 \times 0.940)$ $= 44.129\ 39 + 15.6534$ $= 59.8 \text{ cm}^2$ (to 3 s.f.)
(j)	15×2 $= 30$ cm	15 cm	$5400 \div (3.142 \times 15^2)$ $= 7.6384$ cm $= 7.64$ cm (to 3 s.f.)	0.0054 m^3	$(2 \times 3.142 \times (15)^2)$ $+ (2 \times 3.142 \times 15 \times 7.6384)$ $= 1413.9 + 719.996$ $= 2130 \text{ cm}^2$ (to 3 s.f.)

8. **(i)** Volume of hall

$= \text{volume of cuboid}$
$\quad + \text{volume of half-cylindrical ceiling}$

$= (30 \times 80 \times 10) + \left(\frac{1}{2} \times 3.142 \times \left(\frac{30}{2}\right)^2 \times 80\right)$

$= 24\ 000 + 28\ 278$

$= 52\ 278$

$= 52\ 300 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of hall

$= [2(30 \times 10) + 2(80 \times 10) + (80 \times 30)]$
$\quad + \frac{1}{2}[(2 \times 3.142 \times 15^2) + (2 \times 3.142 \times 15 \times 80)]$

$= [600 + 1600 + 2400] + \frac{1}{2}[1413.9 + 7540.8]$

$= 4600 + 4477.35$

$= 9077.35$

$= 9080 \text{ m}^2$ (to 3 s.f.)

9. Let the breadth of the rectangle be x cm.
Then the length of the rectangle is $2x$ cm.
Circumference of the wire

$= 2 \times 3.142 \times \frac{35}{2}$

$= 3.142 \times 35$

$= 109.97$ cm

Circumference of the wire is the perimeter of the rectangle.

$109.97 = 2(x + 2x)$
$54.985 = x + 2x$
$3x = 54.985$
$x = 18.328\ 33$ (to 5 d.p.)

Area of the rectangle
$= 18.328\ 33 \times 2(18.328\ 33)$
$= 672 \text{ cm}^2$ (to 3 s.f.)

10. **(a)** Perimeter of quadrant $= r + r + \text{arc length } PQ$
$50 = r + r + \text{arc length } PQ$

Arc length $PQ = 50 - r - r$
$= (50 - 2r)$ cm

$\frac{1}{4} \times 2 \times \frac{22}{7} \times r = 50 - 2r$

$\frac{11}{7} \times r = 50 - 2r$

$\frac{11}{7} \times r + 2r = 50$

$3\frac{4}{7}r = 50$

$r = 50 \div 3\frac{4}{7}$

$= 14$ cm

Area of quadrant $= \frac{1}{4} \times \frac{22}{7} \times 14^2 = 154 \text{ cm}^2$

(b) Circumference of wheel

$= 2 \times 3.142 \times \left(\frac{25}{2}\right)$

$= 78.55$ cm

Number of complete revolutions

$= \frac{200}{78.55 \div 100}$

≈ 254.61

$= 254$ revolutions (to 3 s.f.)

Note: The answer cannot be 255 as the wheel has made 254 revolutions but has not yet completed the 255th revolution.

(c) Distance moved by the tip of the hand for 26 minutes

$= \frac{26}{60} \times 2 \times 3.142 \times 8$

$= 21.8$ cm (to 1 d.p.)

(d) Distance travelled in 5 minutes

$= 90 \times \frac{5}{60}$

$= 7.5$ km

Circumference of car wheel
$= 2 \times 3.142 \times 0.000\ 35$
$= 0.002\ 199\ 4$ km

Number of revolutions made
$= 7.5 \div 0.002\ 199\ 4$
$= 3410$ (to 3 s.f.)

(e) Distance covered when the athlete runs round the track once

$= \frac{4}{8}$

$= 0.5$ km

$= 500$ m

Let the radius of the track be r m.
Circumference of the track $= 2 \times 3.142 \times r$
$500 = 2 \times 3.142 \times r$

$\therefore r = \frac{500}{2 \times 3.142}$

$= 79.57$ m (to 2 d.p.)

11. **(a)** Length of arc PR

$= \frac{1}{4} \times 2 \times 3.142 \times 5$

$= 7.855$ cm

Perimeter of the shaded region
$= 5.66 + 7.855 + 3 + (4 + 5) + 4$
$= 29.515$
$= 29.5$ cm (to 3 s.f.)

(b) Area of rectangle
$= 9 \times 8$
$= 72\text{ cm}^2$
Area of $\triangle APQ$
$= \frac{1}{2} \times 4 \times 4$
$= 8\text{ cm}^2$
Area of quadrant BPR
$= \frac{1}{4} \times 3.142 \times 5^2$
$= 19.6375\text{ cm}^2$
Area of shaded region
$= 72 - 8 - 19.6375$
$= 44.3625$
$= 44.4\text{ cm}^2$ (to 3 s.f.)

12. Area of rectangle $ABCD$
$= 60 \times 28$
$= 1680\text{ cm}^2$
Area of semicircle BXC
$= \frac{1}{2} \times 3.142 \times \left(\frac{28}{2}\right)^2$
$= 307.916\text{ cm}^2$
Area of $\triangle ADX$
$= \frac{1}{2} \times 28 \times (60 - 14)$
$= 644\text{ cm}^2$
Area of the shaded region
$= 1680 - 307.916 - 644$
$= 728.084$
$= 728\text{ cm}^2$ (to 3 s.f.)

13. (a) Circumference of the pond
$= 2 \times 3.142 \times 3.2$
$= 20.1088$ m
Circumference of the pond with concrete path
$= 2 \times 3.142 \times (3.2 + 1.4)$
$= 2 \times 3.142 \times 4.6$
$= 28.9064$ m
Perimeter of the shaded region
$= 28.9064 + 20.1088$
$= 49.0152$
$= 49.0$ m (to 3 s.f.)

(b) Area of the pond
$= 3.142 \times 3.2^2$
$= 32.174\ 08\text{ m}^2$
Area of the pond with concrete path
$= 3.142 \times 4.6^2$
$= 66.484\ 72\text{ m}^2$
Area of the shaded region
$= 66.484\ 72 - 32.174\ 08$
$= 34.310\ 64$
$= 34.3\text{ m}^2$ (to 3 s.f.)
$\therefore$ The area of the concrete path is 34.3 m^2.

14. (a) Area of shaded region A
= area of circle with radius 5 cm
$= 3.142 \times 5^2$
$= 78.55$
$= 78.6\text{ cm}^2$ (to 3 s.f.)

(b) Area of circle with radius 10 cm
$= 3.142 \times 10^2$
$= 314.2\text{ cm}^2$
Area of circle with radius 8 cm
$= 3.142 \times 8^2$
$= 201.088\text{ cm}^2$
Area of shaded region B
$= 314.2 - 201.088$
$= 113.112$
$= 113\text{ cm}^2$ (to 3 s.f.)

15. (a) 1.25 m $= 1.25 \times 100$
$= 125$ cm
The largest possible radius is 125.4 cm or 1.254 m.

(b) Smallest possible radius = 124.5 cm = 1.245 m
Smallest possible area
$= 3.142 \times (1.245)^2$
$= 4.870\ 178\ 55$
$= 4.870\text{ m}^2$ (to 4 s.f.)

16. Area of quadrant
$= \frac{1}{4} \times 3.142 \times 21^2$
$= 346.4055\text{ cm}^2$
Area of $\triangle OCA$
$= \frac{1}{2} \times 13 \times 21$
$= 136\frac{1}{2}\text{ cm}^2$
Area of shaded region
$= 346.4055 - 136\frac{1}{2}$
$= 209.9055$
$= 210\text{ cm}^2$ (to 3 s.f.)

17. (a) Area of quadrant
$= \frac{1}{4} \times 3.142 \times 40^2$
$= 1256.8\text{ cm}^2$
Area of triangle
$= \frac{1}{2} \times 40 \times 40$
$= 800\text{ cm}^2$
Area of shaded region
$= 1256.8 - 800$
$= 456.8$
$= 457\text{ cm}^2$ (to 3 s.f.)

(b) Area of square

$= 24 \times 24$

$= 576 \text{ cm}^2$

Area of a circle with radius 12 cm

$= 3.142 \times 12^2$

$= 452.448 \text{ cm}^2$

Area of shaded region

$= 576 - 452.448$

$= 123.552$

$= 124 \text{ cm}^2$ (to 3 s.f.)

(c) Area of semicircle with radius 5 cm

$= \frac{1}{2} \times 3.142 \times 5^2$

$= 39.275 \text{ cm}^2$

Area of triangle

$= \frac{1}{2} \times 6 \times 8$

$= 24 \text{ cm}^2$

Area of shaded region

$= 39.275 - 24$

$= 15.275$

$= 15.3 \text{ cm}^2$ (to 3 s.f.)

(d) Area of trapezium

$= \frac{1}{2}(23 + 33) \times 19$

$= 532 \text{ cm}^2$

Area of triangle

$= \frac{1}{2} \times 33 \times 19$

$= 313.5 \text{ cm}^2$

Area of shaded region

$= 532 - 313.5$

$= 218.5 \text{ cm}^2$

(e) Area of semicircle with diameter 5 cm

$= \frac{1}{2} \times 3.142 \times \left(\frac{5}{2}\right)^2$

$= 9.818\ 75 \text{ cm}^2$

Area of semicircle with diameter 2 cm

$= \frac{1}{2} \times 3.142 \times \left(\frac{2}{2}\right)^2$

$= 1.571 \text{ cm}^2$

Area of semicircle with diameter 3 cm

$= \frac{1}{2} \times 3.142 \times \left(\frac{3}{2}\right)^2$

$= 3.534\ 75 \text{ cm}^2$

Area of shaded region

$= 9.818\ 75 - 1.571 + 3.534\ 75$

$= 11.7825$

$= 11.8 \text{ cm}^2$ (to 3 s.f.)

(f) Area of trapezium

$= \frac{1}{2} \times (19 + 29) \times 21$

$= 504 \text{ cm}^2$

Area of circle with diameter 21 cm

$= 3.142 \times \left(\frac{21}{2}\right)^2$

$= 346.4055 \text{ cm}^2$

Area of shaded region

$= 504 - 346.4055$

$= 157.5945$

$= 158 \text{ cm}^2$ (to 3 s.f.)

(g) Total shaded area

= area of semicircle with radius 12 cm
+ area of rectangle 23 cm by 12 cm
+ area of rectangle 17 cm by 12 cm
+ area of triangle

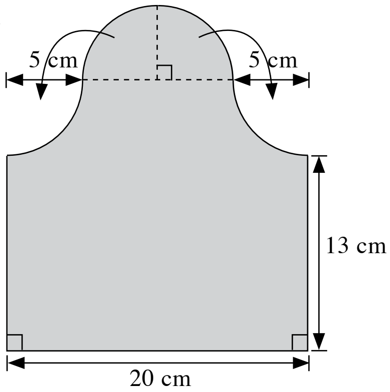
$= \left(\frac{1}{2} \times 3.142 \times 12^2\right) + (12 \times 23) + (17 \times 12)$
$+ \left(\frac{1}{2} \times 6 \times 4\right)$

$= 226.224 + 276 + 204 + 12$

$= 718.224$

$= 718 \text{ cm}^2$ (to 3 s.f.)

(h) Place the quadrants and fill the gap. The figure is then changed into a rectangle with dimensions 20 cm by 18 cm.

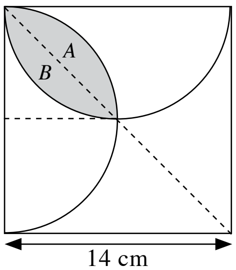


Area of shaded region

= area of rectangle with dimension 20 cm by 18 cm

$= 20 \times 18$

$= 360 \text{ cm}^2$

(i) 

Area of region A

$= \left(\frac{1}{4} \times 3.142 \times 7^2\right) - \left(\frac{1}{2} \times 7 \times 7\right)$

$= 38.4895 - 24.5$

$= 13.9895 \text{ cm}^2$

Area of region B = area of region A

$= 13.9895 \text{ cm}^2$

$\therefore$ Area of shaded region

$= 2 \times 13.9895$

$= 27.979$

$= 28.0 \text{ cm}^2$ (to 3 s.f.)

18. **(a)** Perimeter of semicircle

$= \left(\frac{1}{2} \times 2 \times 3.142 \times \frac{2x}{2}\right) + 2x$

$= 3.142x + 2x$

$= 5.142x$ cm

Perimeter of rectangle

$= 2[(x + 11) + (x - 3)]$

$= 2(x + x + 11 - 3)$

$= 2(2x + 8)$ cm

$5.142x = 2(2x + 8)$

$5.142x = 4x + 16$

$5.142x - 4x = 16$

$1.142x = 16$

$x = 14.0105$

$= 14.0$ (to 3 s.f.)

(b) Area of semicircle

$= \frac{1}{2} \times 3.142 \times \left(\frac{2 \times 14.01}{2}\right)^2$

$= 308.356\,037\,1 \text{ cm}^2$

Length of rectangle $= 14.01 + 11$

$= 25.01$ cm

Breadth of rectangle $= 14.01 - 3$

$= 11.01$ cm

Area of rectangle $= 25.01 \times 11.01$

$= 275.3601 \text{ cm}^2$

Difference in area $= 308.356\,037\,1 - 275.3601$

$= 32.9959$

$= 33.0 \text{ cm}^2$ (to 3 s.f.)

19. **(a)** Let the radius of the semicircle be r cm.

Area of semicircle

$= \frac{1}{2} \times 3.142 \times r^2$

$= 1.571r^2 \text{ cm}^2$

Area of triangle AFE

$= \frac{1}{2} \times 2r \times r$

$= r^2 \text{ cm}^2$

Area of shaded region $= 1.571r^2 - r^2$

$73 = 1.571r^2 - r^2$

$0.571r^2 = 73$

$r^2 = 127.845\,884\,4$

$r = 11.3$ (to 3 s.f.)

Length of $AE = 2 \times 11.306\,895$

$= 22.6138$

$= 22.6$ cm (to 3 s.f.)

(b) Area of trapezium $ABDE$

$= \frac{1}{2} \times (48 + 22.6138) \times 20$

$= 706.138$

$= 706 \text{ cm}^2$ (to 3 s.f.)

20. **(a)** **(i)** Base area $= (8 \times 3) + \frac{1}{2}(3 + 6) \times 4$

$= 24 + 18$

$= 42 \text{ cm}^2$

Volume of prism

= base area × height

$= 42 \times 6$

$= 252 \text{ cm}^3$

(ii) Total surface area

= area of all the surfaces

$= 42 + 42 + 2(6 \times 3) + (5 \times 6) + (6 \times 2)$
$+ (6 \times 7) + (8 \times 6)$

$= 84 + 36 + 30 + 12 + 42 + 48$

$= 252 \text{ cm}^2$

(b) **(i)** Base area $= \frac{1}{2}(9 + 6) \times 4$

$= 30 \text{ cm}^2$

Volume of prism $= 30 \times 8$

$= 240 \text{ cm}^3$

(ii) Total surface area

$= 30 + 30 + (8 \times 9) + (6 \times 8) + (8 \times 5)$
$+ (4 \times 8)$

$= 60 + 72 + 48 + 40 + 32$

$= 252 \text{ cm}^2$

21. (i) Area of face $ABQP$

$= \frac{1}{2}(7 + 13) \times 8$

$= 80 \text{ cm}^2$

(ii) Base area = area of face $ABQP$ = 80 cm^2

Volume of solid

= base area × height

$= 80 \times 40$

$= 3200 \text{ cm}^3$

(iii) Total surface area

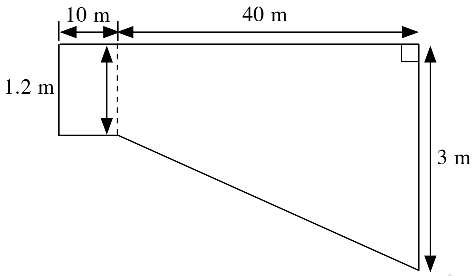
= area of all the faces

$= 2(80) + (13 \times 40) + (7 \times 40) + (8 \times 40) + (10 \times 40)$

$= 160 + 520 + 280 + 320 + 400$

$= 1680 \text{ cm}^2$

22. A drawing of the cross-section of the swimming pool is helpful in solving the problem.



Area of cross-section

$= \frac{1}{2} \times (1.2 + 3) \times 40 + 10 \times 1.2$

$= 96 \text{ m}^2$

Volume of water in the pool when it is full

= area of cross-section × width

$= 96 \times 32$

$= 3072 \text{ cm}^3$

23. (i) Let the radius of the base of the cylinder be r cm.

Circumference of base of cylinder = $2\pi r$

$88 = 2 \times 3.142 \times r$

$r = 14.004$

Total surface area

$= 2\pi r^2 + (\text{circumference} \times \text{height})$

$= (2 \times 3.142 \times 14.004^2) + (88 \times 10)$

$= 1232.367\ 909 + 880$

$= 2112.367\ 909$

$= 2110 \text{ cm}^2$ (to 3 s.f.)

(ii) Volume of cylinder

$= \pi r^2 h$

$= 3.142 \times 14.004^2 \times 10$

$= 6161.840$

$= 6160 \text{ cm}^3$ (to 3 s.f.)

24. Volume of water in container P

$= 3.142 \times \left(\frac{3}{2}\right)^2 \times 24$

$= 169.668 \text{ cm}^3$

Let the height of water in container Q be h cm.

Volume of water in container Q = 169.668 cm^3

Base area of container Q

$= 3.142 \times \left(\frac{8}{2}\right)^2$

$= 50.272 \text{ cm}^2$

$50.272 \times h = 169.668$

$h = 3\frac{3}{8}$

The height of water in container Q is $3\frac{3}{8}$ cm.

25. Volume of water in the cylinder when it is filled to the brim

$= 3.142 \times \left(\frac{10}{2}\right)^2 \times 30$

$= 2356.5 \text{ cm}^3$

Volume of water in the tank before the ball bearings are added

$= \frac{3}{8} \times 2356.5$

$= 883.6875 \text{ cm}^3$

Volume of water and ball bearings

$= \frac{1}{2} \times 2356.5$

$= 1178.25 \text{ cm}^3$

Volume of 8 ball bearings

$= 1178.25 - 883.6875$

$= 294.5625 \text{ cm}^3$

Volume of each ball bearing

$= 294.5625 \div 8$

$= 36.820$

$= 36.8 \text{ cm}^3$ (to 3 s.f.)

26. (i) Total surface area of an open cylinder

$= \pi r^2 + 2\pi rh$

$= (3.142 \times 14^2) + (2 \times 3.142 \times 14 \times 30)$

$= 615.832 + 2639.28$

$= 3255.112$

$= 3260 \text{ cm}^2$ (to 3 s.f.)

(ii) $1 \text{ m}^2 = 1 \times 100 \times 100 = 10\ 000 \text{ cm}^2$

$10\ 000 \text{ cm}^2$ costs 750 paisas

3255.112 cm^2 costs 244.1334 paisas

= 244 paisas (to the nearest paisa)

27. (a) Volume of metal cube = $(46)^3$

$= 97\ 336 \text{ cm}^3$

Volume of each cylindrical rod

$= 3.142 \times 2^2 \times 3.2$

$= 40.2176 \text{ cm}^3$

Maximum number of rods that can be obtained

$= \dfrac{97\,336}{40.2176}$

≈ 2420

(b) Volume of the metal disc

$= 3.142 \times 8^2 \times 3$

$= 603.264$ cm^3

Volume of each bar

$= 3.142 \times 1^2 \times 4.2$

$= 13.1964$ cm^3

Maximum number of bars that can be obtained

$= \dfrac{603.264}{13.1964}$

$= 45.7143$

≈ 45

(c) Volume of butter

$= 3.142 \times 3^2 \times 10$

$= 282.78$ cm^3

Volume of each circular disc

$= 3.142 \times (1.5)^2 \times 0.8$

$= 5.6556$ cm^3

Maximum number of discs formed

$= \dfrac{282.78}{5.6556}$

$= 50$

28. Volume of the metal

$= 12 \times 18 \times 10$

$= 2160$ cm^3

Volume of each cylindrical plate

$= 2160 \div 45$

$= 48$ cm^3

Let the thickness of each plate be t cm.

$48 = 3.142 \times 1.2^2 \times t$

$t = 10.61$ cm (to 2 d.p.)

$\therefore$ The thickness of each plate is 10.61 cm.

29. (i) Internal curved surface area

$= 2\pi rh$

$= 2 \times 3.142 \times 9 \times (12 \times 100)$

$= 67\,867.2$ cm^2

$= 6.79$ m^2 (to 3 s.f.)

(ii) External radius of the pipe $= 9 + 0.5$

$= 9.5$ cm

Volume of metal

$= [3.142 \times (9.5)^2 \times 1200] - (3.142 \times 9^2 \times 1200)$

$= [3.142 \times 1200](9.5^2 - 9^2)$

$= 34\,876.2$

$= 34\,900$ cm^3 (to 3 s.f.)

30. (i) Convert 385 litres to cm^3.

385 litres $= 385 \times 1000 = 385\,000$ cm^3

(ii) Base area $= 3.142 \times (70)^2$

$= 15\,395.8$ cm^2

Volume of water in tank = base area × height h

$385\,000 = 15\,395.8 \times h$

$h = 25.007$

$= 25.0$ cm (to 3 s.f.)

(iii) Total surface area of the liquid in contact with the cylindrical tank

$= (3.142 \times 70^2) + (2 \times 3.142 \times 70 \times 25.007)$

$= 15\,395.8 + 11\,000.08$

$= 26\,395.88$

$= 26\,400$ cm^2 (to 3 s.f.)

31. (a) Since water is discharged through the pipe at a rate of 28 m/min, the volume of water discharged in 1 minute is the volume of water that fills the pipe to a length of 28 m.

In 1 minute, volume of water discharged

= volume of pipe of length 28 m

$= \pi r^2 h$

$= 3.142 \times (4.2 \div 100)^2 \times 28$

$= 0.155\,189\,664$ cm^3

Volume of water in rectangular tank

$= 4 \times 2.5 \times 2.4$

$= 24$ m^3

Amount of time needed to fill the tank completely

$= \dfrac{24}{0.155\,189\,664}$

$= 154.6495$ minutes

= 2 hours and 35 minutes (to the nearest minute)

(b) Since water is discharged through the pipe at a rate of 3.4 m/s, the volume of water discharged in 1 second is the volume of water that fills the pipe to a length of 3.4 m.

In 1 second, volume of water discharged

= volume of pipe of length 3.4 m

$= \pi r^2 h$

$= 3.142 \times [(5.2 \div 2) \div 100]^2 \times 3.4$

$= 0.007\,221\,572\,8$ m^3

Volume of cylindrical tank

$= 3.142 \times (2.3)^2 \times 1.6$

$= 26.593\,888\,8$ m^3

Amount of time needed to fill the tank

$= 26.593\,888\,8 \div 0.007\,221\,572\,8$

$= 3682.561\,893$ seconds

= 61 minutes (to the nearest minute)

(c) Base area of trapezium

$= \frac{1}{2} \times (7 + 5) \times 2.5$

$= 15 \text{ m}^2$

In 1 hour, volume of water discharged

$= 15 \times (12 \times 1000)$

$= 180\,000 \text{ m}^3$

In 1 second,

volume of water discharged

$= (180\,000 \div 3600)$

$= 50 \text{ m}^3$

In 5 seconds, the volume of water discharged

$= 5 \times 50 = 250 \text{ m}^3$

(d) Since water is discharged through the pipe at a rate of 18 km/h, the volume of water discharged in 1 hour is the volume of water that fills the pipe to a length of 18 km = 18 000 m.

In 1 hour, volume of water discharged

= volume of pipe of length 18 km

$= 3.142 \times (4 \div 100)^2 \times 18\,000$

$= 90.4896 \text{ m}^3$

In $1\frac{2}{3}$ hours, the volume of water discharged

$= 1\frac{2}{3} \times 90.4896$

$= 150.816 \text{ m}^3$

Volume of swimming pool $= 50 \times 25 \times$ height h

$150.816 = 1250 \times h$

$\therefore h = 0.120\,652\,8$ m

$= 12.1$ cm (to 3 s.f.)

32. (a) Convert 10.5 kg to g.

$10.5 \text{ kg} = 10.5 \times 1000 = 10\,500$ g

Volume of metal

$= \frac{10\,500}{3.5}$

$= 3000 \text{ cm}^3$

$3000 = 4 \times 3 \times x$

$x = 250$

(b) Convert 22.44 kg to g.

$22.44 \text{ kg} = 22.44 \times 1000 = 22\,440$ g

Volume of metal

$= \frac{22\,440}{13.6}$

$= 1650 \text{ cm}^3$

Let the radius of the glass cylinder be x cm.

$1650 = 3.142 \times x^2 \times 21$

$x^2 = 25.0068$

$x = 5.000\,68$

Diameter of the glass cylinder

$= 2 \times 5.000\,68$

$= 10.0$ cm (to 3 s.f.)

33. (i) Base area $= \frac{1}{2} \times (12 + 8) \times 7$

$= 70 \text{ cm}^2$

Volume of block = base area × length

$= 70 \times 28$

$= 1960 \text{ cm}^3$

(ii) Total surface area of block

$= 70 + 70 + (12 \times 28) + (7 \times 28) + (28 \times 8.06) + (8 \times 28)$

$= 70 + 70 + 336 + 196 + 225.68 + 224$

$= 1121.68 \text{ cm}^2$

34. (i) Total surface area of the solid block

$= (2 \times 3.142 \times 14^2) + (2 \times 3.142 \times 14 \times [1.2 \times 100])$

$= 1231.664 + 10\,557.12$

$= 11\,788.784$

$= 11\,800 \text{ cm}^2$ (to 3 s.f.)

(ii) Volume of block

$= 3.142 \times (14)^2 \times (1.2 \times 100)$

$= 73\,899.84$

$= 73\,900 \text{ cm}^3$ (to 3 s.f.)

35. (i) Volume of box

$= 48 \times 36 \times 15$

$= 25\,920 \text{ cm}^3$

Number of items in box

$= (48 \div 7) \times (36 \div 7) \times (15 \div 7)$

$\approx 6 \times 5 \times 2$

$= 60$

Total volume of items $= 60 \times 7^3$

$= 20\,580 \text{ cm}^3$

Volume of sawdust

$= 25\,920 - 20\,580$

$= 5340 \text{ cm}^3$

(ii) Mass of sawdust = density × volume

$= 0.75 \times 5340$

$= 4005$ g

36. (i) Total surface area of the cuboid

$= 2[(30 \times 25) + (30 \times 15) + (25 \times 15)]$

$= 2[750 + 450 + 375]$

$= 3150 \text{ cm}^2$

(ii) Volume of cuboid

$= 30 \times 25 \times 15$

$= 11\,250 \text{ cm}^3$

Volume of each coin

$= 3.142 \times 1.5^2 \times (2.4 \div 10)$

$= 1.696\,68 \text{ cm}^3$

Number of coins that can be made
= 11 250 ÷ 1.696 68
≈ 6630
(Note: The answer is not 6631 as the number of coins is 6630.6, which is less than 6631.)

(iii) Total volume of coins
= 6630 × 1.696 68
= 11 248.9884 cm^3
Volume of molten metal left behind
= 11 250 – 11 248.9884
= 1.0116
= 1.01 cm^3 (to 3 s.f.)

37. (i) Convert 3780 litres to m^3.
3780 l = (3780 × 1000) ÷ 100 ÷ 100 ÷ 100
= 3.78 m^3
Let the depth of the liquid in the tank be d m.
$3.78 = 4.2 \times 1.8 \times d$
$\therefore d = 0.5$
The depth of the liquid in the tank is 0.5 m or 50 cm.

(ii) Volume of increase in liquid level
= 420 × 180 × (1.6)
= 120 960 cm^3
Volume of one solid brick
$= \dfrac{120\,960}{380}$
= 318.315 789 5
= 318 cm^3 (to 3 s.f.)

38. (i) Volume of closed container
= volume of cuboid + volume of half – cylinder
$= (28 \times 60 \times 40) + \left(\frac{1}{2} \times 3.142 \times \left(\frac{28}{2}\right)^2 \times 60\right)$
= 67 200 + 18 474.96
= 85 674.96 cm^3
= (85 674.96 ÷ 1000) litres
= 85.7 litres (to 3 s.f.)

(ii) Total surface area of the container
= surface area of the cuboid (without the top surface)
+ surface area of the half cylinder
$= 2 \times \left[28 \times 40 + \frac{1}{2} \times 3.142 \times 14^2\right]$
$+ 2(60 \times 40) + (28 \times 60)$
$+ \left(\frac{1}{2} \times 2 \times 3.142 \times 14 \times 60\right)$
= 2[1120 + 307.916] + 4800 + 1680 + 2639.28
= 2855.832 + 4800 + 1680 + 2639.28
= 11 975.112 cm^2
= 1.197 511 2 m^2
= 1.20 m^2 (to 3 s.f.)

39. Volume of two cylindrical discs
$= 2\left[\pi \times \left(\frac{120}{2}\right)^2 \times 12\right]$
= 86 400π cm^3
Volume of the connecting cylinder of diameter 40 cm
$= \pi \times \left(\frac{40}{2}\right)^2 \times (94 - 12 - 12)$
= 28 000π cm^3
Total volume of drum (before the cylinder of diameter of 16 cm is removed)
= (86 400 + 28 000)π
= 114 400π cm^3
Volume of cylinder, of diameter 16 cm, removed from the drum
$= \pi \times \left(\frac{16}{2}\right)^2 \times 94$
= 6016π cm^3
Volume of wood used to make the drum
= (114 400 – 6016)π
= 108 384π cm^3
= 108 000π cm^3 (to 3 s.f.)

40. (i) Volume of cylindrical container
= 3.142 × $(14)^2$ × 40
= 24 633.28
= 24 600 cm^3 (to 3 s.f.)

(ii) Surface area of one cylindrical container
= (2 × 3.142 × 14 × 40) + (2 × 3.142 × 14^2)
= 4750.704 cm^2
Surface area of 450 cylinders
= 450 × 4750.704
= 2 137 816.8 cm^2
4200 cm^2 surface requires 0.24 litres of paint.
2 137 816.8 cm^2 requires 122.160 96 litres of paint.
123 litres of paint must be purchased to paint all 450 containers.
Cost to paint the containers
= PKR 8.70 × 123
= PKR 1070.10

41. (i) Volume of the rectangular block

$= 12 \times 18 \times 10$

$= 2160 \text{ cm}^3$

Volume of cylinder removed from the block

$= 3.142 \times \left(\frac{7}{2}\right)^2 \times 18$

$= 692.811 \text{ cm}^3$

Volume of remaining solid

$= 2160 - 692.811$

$= 1467.189$

$= 1470 \text{ cm}^3$ (to 3 s.f.)

(ii) Total surface area of the remaining solid

$= 2\left[12 \times 10 - \left(3.142 \times \left(\frac{7}{2}\right)^2\right)\right]$

$\quad + 2[(18 \times 12) + (18 \times 10)]$

$= 2[120 - 38.4895] + 2[396]$

$= 955.021$

$= 955 \text{ cm}^2$ (to 3 s.f.)

Advanced

42. Let the radius of each circle be r cm.

Area of each circle $= \pi r^2$

$36\pi = \pi r^2$

$r^2 = 36$

$\therefore r = \sqrt{36} = 6$

Length of $CD = 6 + 6$

$= 12$ cm

43. (i) Let the radius of cylinder A be r cm and the height be h cm.

Volume of cylinder A

$= \pi r^2 h$

$= 343 \text{ cm}^3$

Then the radius of cylinder B will be $\frac{1}{3} r$ cm and the height will be h cm.

Volume of cylinder B

$= \pi \times \left(\frac{1}{3} r\right)^2 \times h$

$= \pi \times \frac{1}{9} \times r^2 \times h$

$= \frac{1}{9} \times \pi r^2 h$

$= \frac{1}{9} \times 343$

$= 38.111$

$= 38.1 \text{ cm}^3$ (to 3 s.f.)

(ii) Number of cubes formed

$= \frac{38.111}{2^3}$

$= 4.763\ldots$

The maximum number of cubes formed is 4.

44. Total thickness of the paper towel after it is being rolled

$= (1 \div 10) \times 90$

$= 9$ cm

Total radius of the paper towel and roll

$= 9 + 2.5$

$= 11.5$ cm

Base area of the paper towel, in the form of a cylinder

$= \frac{22}{7} \times [(11.5)^2 - (2.5)^2]$

$= 396 \text{ cm}^2$

Volume of paper towel

$=$ base area $\times$ width of towel

$= 396 \times 14$

$= 5544$

$= 5540 \text{ cm}^3$ (to 3 s.f.)

New Trend

45. (a) $A = \pi\left(\frac{5r + 5kr}{2}\right)^2$

$= \pi\left[\frac{5r(1+k)}{2}\right]^2$

$= \frac{25}{4}\pi r^2(1+k)^2$

(b) When $k = 3$,

Area of large circle $= \frac{25}{4}\pi r^2(4)^2$

$= 100\pi r^2 \text{ cm}^2$

Area of shaded region

$= \frac{100 \sim r^2}{2} + \frac{1}{2}\pi\left(\frac{5kr}{2}\right)^2 - \frac{1}{2}\pi\left(\frac{5r}{2}\right)^2$

$= 50\pi r^2 + \frac{1}{2}\pi\left(\frac{225r^2}{4}\right) - \frac{1}{2}\pi\left(\frac{25r^2}{4}\right)$

$= 50\pi r^2 + \frac{225 \sim r^2}{8} - \frac{25 \sim r^2}{8}$

$= 75\pi r^2 \text{ cm}^2$

Area of unshaded region

$= \frac{100 \sim r^2}{2} + \frac{1}{2}\pi\left(\frac{5r}{2}\right)^2 - \frac{1}{2}\pi\left(\frac{5kr}{2}\right)^2$

$= 50\pi r^2 + \frac{25 \sim r^2}{8} - \frac{225 \sim r^2}{8}$

$= 50\pi r^2 - 25\pi r^2$

$= 25\pi r^2 \text{ cm}^2$

Difference in area $= 75\pi r^2 - 25\pi r^2$

$= 50\pi r^2 \text{ cm}^2$

46. Surface area of cross-section

$= \pi\left(\frac{1.3}{2}\right)^2 + \frac{1}{2} \times 2.6 \times 2.25 - \pi\left(\frac{0.5}{2}\right)^2$

$= 0.4225\pi + 2.925 - 0.0625\pi$

$= (0.36\pi + 2.925)\ \text{cm}^2$

Volume of platinum $= 0.2(0.36\pi + 2.925)$

$= (0.072\pi + 0.585)\ \text{cm}^3$

Price of platinum used $= 21.5(0.072\pi + 0.585) \times$ PKR 43.48

= PKR 758.3210 (to 4 d.p.)

Total value of pendant = PKR 4200 + PKR 758.3210

= PKR 4958.32 (to the nearest paisa)

47. Convert 100 litres to cm^3.

$100\ l = 100 \times 1000 = 100\ 000\ \text{cm}^3$

Volume of tank = cross-sectional area × height

Height = volume of tank ÷ cross-sectional area

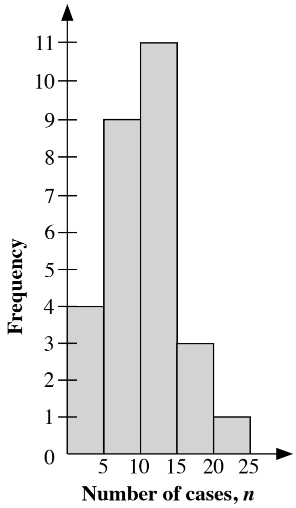
$= 100\ 000 \div \pi(30)^2$

= 35.4 cm (to 3 s.f.)

Chapter 13 Averages of Statistical Data

Basic

1. (i)



(ii) Number of days = 3 + 1

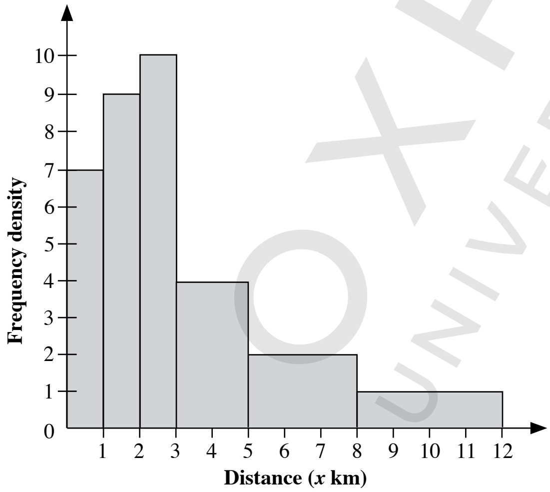
= 4

2. (a) The 4th day had the greatest number of employees report sick. 35 workers reported sick.

(b) The 10th day had the least number of employees report sick. 13 workers reported sick.

(c) The number of employees who reported sick was more than 30 on the 4th and 8th day.

3.



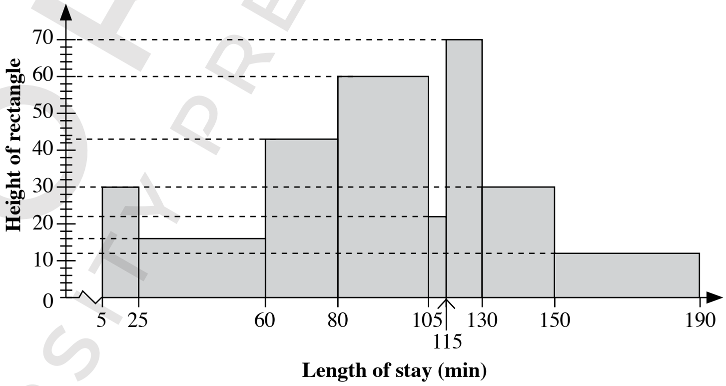
5. (a) Total number of cars

= 60 + 56 + 86 + 150 + 60 + 105 + 60 + 48

= 625

(b) Since the class intervals are unequal, the histogram is to be drawn using either height of rectangle or frequency density.

Class interval	Class width		Frequency	Rectangle's height
5 – 24	20	2 × standard	60	60 ÷ 2 = 30
25 – 59	35	3.5 × standard	56	56 ÷ 3.5 = 16
60 – 79	20	2 × standard	86	86 ÷ 3 = 43
80 – 104	25	2.5 × standard	150	150 ÷ 2.5 = 60
105 – 114	10	1 × standard	60	60 ÷ 1 = 22
115 – 129	15	1.5 × standard	105	105 ÷ 1.5 = 70
130 – 149	20	2 × standard	60	60 ÷ 2 = 30
150 – 189	40	4 × standard	48	48 ÷ 4 = 12



Intermediate

6. (i)

Age of patient, *x* years	Frequency
$10 \leqslant x < 20$	85
$20 \leqslant x < 30$	117
$30 \leqslant x < 40$	38
$40 \leqslant x < 50$	24
$50 \leqslant x < 60$	18
$60 \leqslant x < 70$	16
Total frequency	300

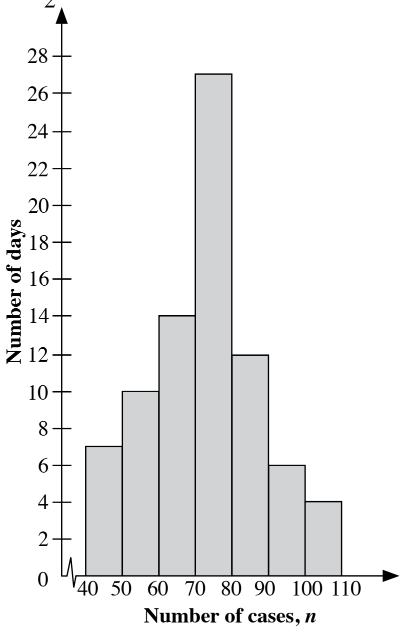
(ii) Percentage of patients who are at least 50 years old

$= \dfrac{18 + 16}{300} \times 100\%$

= 11.3% (to 3 s.f.)

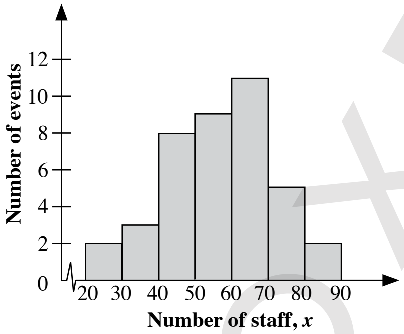
(iii) No. The actual ages of the patients in the interval $20 \leqslant x < 30$ are not known, so it is incorrect for Sarah to assume that al the patients in this interval are $\frac{20 + 30}{2} = 25$ years old.

7. (i)



(ii) No, the most number of cases occur in the interval $70 \leqslant n < 80$, but it is not correct to take the mid-value of this interval.

8. (i)



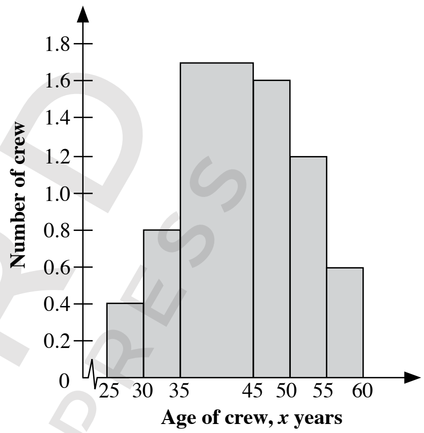
(ii) Number of events $= 45\% \times 40$

$= 18$

$\therefore p = 60$

9. (i)

Age of crew, x years	Frequency	Frequency density
$25 \leqslant x < 30$	2	0.4
$30 \leqslant x < 35$	4	0.8
$35 \leqslant x < 45$	17	1.7
$45 \leqslant x < 50$	8	1.6
$50 \leqslant x < 55$	6	1.2
$55 \leqslant x < 60$	3	0.6



(ii) Number of crew $= 0.85 \times 40$

$= 34$

$\therefore p = 35$

10. (i) Mean mass $\approx \dfrac{32 \times 20 + 38 \times 35 + 64 \times 45 + 35 \times 55 + 22 \times 65 + 9 \times 85}{200}$

$= 44.85$ kg

(ii) Probability that the steel bar requires another transportation vehicle $= \dfrac{9}{200}$

11. (a) Mean amount of medical claims

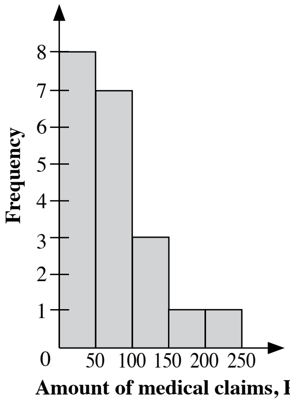
$= \dfrac{150 + 44 + 225 + \ldots + 77 + 55 + 136}{20}$

$=$ PKR 70.40

(b)

Amount of medical claims, PKR m	Frequency
$0 \leqslant m < 50$	8
$50 \leqslant m < 100$	7
$100 \leqslant m < 150$	3
$150 \leqslant m < 200$	1
$200 \leqslant m < 250$	1
Total frequency	20

(c) (i)



(ii) Estimate for the mean amount of medical claims

$$= \frac{8 \times 25 + 7 \times 75 + 3 \times 125 + 1 \times 175 + 1 \times 225}{20}$$

= PKR 75

(d) There is a difference of PKR 4.60 in the answers in **(a)** and **(c)(ii)**. The mean amount calculated in **(a)** is the exact value as it is based on the individual values, but the mean amount calculated in **(c)(ii)** is an estimate as the mid-values of each interval are used.

12. Total number of vehicles along Section A = 50

The median average speed along Section A lies in the interval $60 \leqslant v < 70$.

Total number of vehicles along Section B = 49

The median average speed along Section B lies in the interval $70 \leqslant v < 80$.

As the actual data in these intervals is not known, it is incorrect for Ethan to obtain the median average speed along Section A by taking $\frac{60 + 70}{2}$ = 65 km/h or to obtain the median average speed along Section B by taking $\frac{70 + 80}{2}$ = 75 km/h.

Chapter 14 Probability

Basic

1. **(i)** Probability of drawing a King $= \frac{4}{52}$

$= \frac{1}{13}$

(ii) Probability of drawing the King of diamonds $= \frac{1}{52}$

(iii) Probability of drawing a heart $= \frac{13}{52}$

$= \frac{1}{4}$

(iv) Probability of drawing a picture card $= \frac{12}{52}$

$= \frac{3}{13}$

2. **(a)** **(i)** Probability that the customer wins PKR 88 cash

$= \frac{1}{8}$

(ii) Probability that the customer wins a PKR 10 shopping

voucher $= \frac{3}{8}$

(iii) Probability that the customer wins a packet of dried scallops = 0

(b) A pair of movie tickets and a can of abalone

3. **(i)** Angle corresponding to the sector representing beans

$= 360° - 150° - 90° - 50°$

$= 70°$

Probability that the student prefers beans $= \frac{70°}{360°}$

$= \frac{7}{36}$

(ii) Probability that the student prefers broccoli or carrots

$= \frac{90° + 50°}{360°}$

$= \frac{140°}{360°}$

$= \frac{7}{18}$

4. **(i)** Probability that a bag selected has a mass of exactly

1 kg $= 1 - \frac{1}{40} - \frac{1}{160}$

$= \frac{31}{32}$

(ii) Number of bags each with a mass of less than 1 kg

$= \frac{1}{160} \times 8000$

$= 50$

Intermediate

5. Probability that it is labelled Gold $= 1 - \frac{1}{5} - \frac{1}{4}$

$= \frac{11}{20}$

Total number of boxes $= 55 \div \frac{11}{20}$

$= 100$

6. **(a)** **(i)** Probability of selecting a vowel $= \frac{1}{7}$

(ii) Probability of selecting a card that bears the letter

C $= \frac{3}{7}$

(b) $\frac{3}{7 + x} = \frac{1}{7}$

$21 = 7 + x$

$x = 14$

7. **(a)** **(i)** Probability that the mark is less than 44

$= \frac{8}{15}$

(ii) Probability that the mark is not a prime number

$= \frac{14}{15}$

(iii) Probability that the mark is divisible by 11

$= \frac{3}{15}$

$= \frac{1}{5}$

(b) Probability that the student obtained the badge

$= \frac{9}{15}$

$= \frac{3}{5}$

(c) Probability that the mark was 39 $= \frac{2}{6}$

$= \frac{1}{3}$

8. **(i)** $\frac{x}{35 + x} = \frac{1}{6}$

$6x = 35 + x$

$5x = 35$

$x = 7$

(ii) Probability of selecting a sports car $= \frac{35 + 5}{35 + 7 + 5}$

$= \frac{40}{47}$

Advanced

9. $$\frac{12 + x + 2}{36 + 12 + 2x + x + 2} = 0.3$$

$$\frac{x + 14}{3x + 50} = 0.3$$

$$x + 14 = 0.9x + 15$$

$$0.1x = 1$$

$$x = 10$$

10. (i) $$\frac{x}{18 + x} = \frac{3}{5}$$

$$5x = 54 + 3x$$

$$2x = 54$$

$$x = 27$$

(ii) Probability of selecting a pink sweet

$$= \frac{15}{18 + 27 + 10 + 15}$$

$$= \frac{15}{70}$$

$$= \frac{3}{14}$$

New Trend

11. (i)

	Smoke	Do not smoke	Total
Male	18	42	60
Female	8	32	40
Total	26	74	100

(ii) Probability that a randomly selected smoker is male

$$= \frac{18}{26}$$

$$= \frac{9}{13}$$

(iii) The respondents of this online survey may not be a good representation of the country's population.